Transmission Network Investment With Distributed Energy Resources and Distributionally Robust Security

Diego Alvarado, Alexandre Moreira, *Student Member, IEEE*, Rodrigo Moreno, *Member, IEEE*, and Goran Strbac, *Member, IEEE*

***Abstract*—Distributed energy resources (DER) have the potential to significantly contribute to network security and hence release latent capacity of existing transmission assets. In this context, we propose a distributionally robust approach to network security in order to recognize the limited data and knowledge associated with the underlying process behind the realization of system contingencies within the transmission expansion planning (TEP) problem, and thus determine the optimal portfolio of DER services necessary to displace, in a secure fashion, inefficient network investments. To do so, we propose a two-stage optimization model where the first stage determines the transmission expansion plan and the scheduling of DER post-contingency services in coordination with further corrective control measures such as generation reserves. The second stage minimizes the expected cost of corrective actions under various contingencies. Through various case studies, we demonstrate the benefits of security services provided by DER and the advantages of our proposed distributionally robust approach (where outage rates are assumed ambiguous) against alternative $n - K$ security and stochastic approaches, where outage rates are either ignored or assumed fully known, respectively.**

***Index Terms*—Transmission expansion planning, distributed energy resources, network security, power systems economics.**

Manuscript received March 1, 2018; revised July 3, 2018; accepted August 13, 2018. Date of publication August 27, 2018; date of current version October 24, 2019. This work was supported by Conicyt, Chile under Grant Fondecyt/1181928 and the Engineering and Physical Sciences Research Council, U.K. under Grants EP/K002252/1 and EP/N005996/1. This work was also supported in part by the Supercomputing Infrastructure of the NLHPC (ECM-02). The work of D. Alvarado was supported by Conicyt—Chile Project PFCHA/MagísterNacional/2017-22171423. The work of A. Moreira was supported by CNPq—Brazil under the Science without Borders scholarship program, Grant 203274/2014-8. The work of R. Moreno was supported by the Complex Engineering Systems Institute (CONICYT-PIA-FB0816; ICM P-05-004-F), and the Conicyt-Chile Grants Newton-Picarte/MR/N026721/1, and SERC Fondap/15110019. Paper no. TPWRS-00299-2018. *(Corresponding author: Rodrigo Moreno.)*

D. Alvarado is with the Department of Electrical Engineering, University of Chile, Santiago 8370451, Chile (e-mail: diego.alvarado@ing.uchile.cl).

A. Moreira and G. Strbac are with the Department of Electrical and Electronic Engineering, Imperial College, London SW7 2AZ, U.K. (e-mail: a.moreira14@imperial.ac.uk; g.strbac@imperial.ac.uk).

R. Moreno is with the Department of Electrical Engineering, University of Chile, Santiago, Chile, and also with the Department of Electrical and Electronic Engineering, Imperial College, London SW7 2AZ, U.K. (e-mail: rmorenovieyra@ing.uchile.cl).

Digital Object Identifier 10.1109/TPWRS.2018.2867226

I. Introduction

The need for transmission network infrastructure in reality is mainly driven by both increasing the economic efficiency and the reliability of electricity systems [1]. In fact, new network investments can present significant benefits in terms of both reduced operational costs and/or unsupplied energy cost. In terms of the latter (i.e. reliability benefits), networks are designed with a certain level of redundancy in order to securely deal with network (or other system) outages without curtailing (high volumes of) demand, which may be costly. According to empirical evidence [1], the application of reliability criteria has been the most important and predominant reason to undertake network investments within system operators' jurisdictions. It is, therefore, paramount to determine the real cost of demand response as an alternative to further network investments.

Additionally, it is envisaged that network security, which has been historically delivered through asset redundancy, should evolve and hence be provided by emerging and innovative non-network technologies, especially those at the demand side in order to release latent network capacity and thus make more efficient use of the existing assets. In this vein, advanced technologies for post-contingency control, which can take advantage of a range of distributed energy resources (DER), can effectively provide security services and thus displace the need for redundant network capacity. The set of DER includes distributed generation (DG), backup generation, an array of storage technologies and demand itself (by utilizing the inherent demand-side flexibility, particularly from non-essential loads and demand associated with the heat and transport sectors). References [2]–[5] report on the possible contributions from DER to the security of supply of the main system.

Within the context of transmission expansion planning, outages have been considered either in a deterministic manner [6]–[9] or in a probabilistic/stochastic fashion [10]–[13]. When the deterministic approach is chosen, likelihood of generation and network outages is ignored. Such likelihood is key to balance pre- and post-contingency costs and therefore determine the right portfolio of network assets, generation reserves, and DER participation in the provision of security. On the other hand, probabilistic/stochastic approaches have been proposed to properly balance pre- and post-contingency generation, network and demand-side measures (by minimizing cost of network investment and expected cost of operation, including demand shedding/curtailment). However stochastic approaches assume perfect information of reliability data, which may be impractical. A more comprehensive review and discussion on deterministic and/versus probabilistic treatment of system security in network planning problems can be found in [14].

Within the context of robust optimization (RO), conservativeness has always been a matter of concern. The first approach presented by [15] proposed to deal with uncertainty by means of box constraints, which led to over-conservative solutions. In [16], ellipsoidal uncertainty is considered to alleviate conservativeness at the expense of sacrificing the benefits of linearity. In order to propose a less conservative approach with linear robust counterparts, the work in [17] has developed a methodology that guarantees feasibility for a predefined number of coefficient changes. In addition, adjustable robust optimization (ARO) [18] has been proposed as a way to consider recourse decisions within the RO framework. Over the years, multiple problems have been addressed via robust optimization and several important theoretical results in RO have been derived [19]. Within power systems literature, some of the many relevant contributions are [20]–[23] for operations and [6], [24]–[26] for expansion planning.

More recently, distributionally robust optimization (DRO) has been attracting a great deal of attention. Unlike stochastic optimization (SO), DRO does not assume full knowledge of the underlying process behind the realization of uncertainty. Nevertheless, DRO can properly take advantage of available moment information. From a theoretical point of view, some of the main contributions are [27]–[29]. In addition, the unit commitment problem has been addressed via DRO approaches in [30]–[32].

In this context, we propose a distributionally robust security framework to incorporate (the right portfolio of) post-contingency DER services in transmission expansion planning (TEP) problems. Due to the distributionally robust nature of our model, it assumes limited knowledge of the underlying process behind the realization of system contingencies, recognizing actual information levels available in reality regarding reliability data (i.e. data-driven approach). Hence, our model can determine the actual levels of network investment that DER services can displace in an economic and secure fashion. While the use of distributionally robust approaches to uncertainty has been already proposed to analyze various problems in power system operation (see [30], [32]–[34]) and investment planning (see [35] and [36]), our model is, to our knowledge, the first one that proposes a distributionally robust approach to network security for determining the right portfolio of DER services in transmission planning. Note that our model has been designed from the transmission network planner's perspective and thus it selects the right portfolio of DER services among those being offered by aggregators. As transmission network operators and planners have no jurisdiction over distribution networks, DER sizing is out of the scope of this paper. Overall, the main contributions of this paper are:

1) Develop a distributionally robust, 2-stage optimization model for the treatment of network security in TEP problems that appropriately captures the participation of DER in the provision of network security.
2) Develop an efficient solution method based on Benders decomposition techniques that determines the optimal solution of the proposed distributionally robust, 2-stage optimization model.
3) Demonstrate that $n-1$ security approaches significantly undermine the value of DER in displacing network investments and generation reserves.
4) Demonstrate that alternative stochastic approaches (hereafter called fixed probabilities approaches) are optimistic regarding the value of DER in displacing network investments and generation reserves.

The remainder of the paper is organized as follows. Section II presents an overview of the proposed framework and the mathematical formulation of the proposed model. Section III describes the proposed solution methodology. In Section IV and V, we present case studies, and finally in Section VI we conclude.

II. Mathematical Formulation

A. *Overview*

The model proposed in this paper aims to determine the optimal set of transmission network investments by balancing the costs of network investments against the corresponding costs of network operation pre- and post-fault, including the costs of network congestions, generation reserves, demand and generation curtailments through special protection schemes (SPS) and, importantly, the costs of an array of DER post-contingency services. In this context, Fig. 1 illustrates that there are several alternatives to network investment in order to increase secured power transfers during a pre-fault condition, comprising utilization of DER (to increase flexible demand in the exporting area and reduce flexible demand in the importing area in case, for instance, a line fault occurs), SPS (to curtail generation in the exporting area and demand in the importing area in case a line fault occurs), and even generation reserves (to reschedule generation post-fault and accommodate power transfers in case a line fault occurs). Furthermore, doing nothing is also an option if the cost of congestion (i.e. cost of network operation without the increase in power transfers pre-fault, that can include the cost of wind spillage) is proved very small.

Note that although the expansion of the transmission network has to be undertaken, in general terms, due to the increasing connections of renewable generation in exporting areas and demand resources in importing areas (for example, from the transport and heat sectors), the investment levels needed to deal with such transmission expansion can be alleviated if the right portfolio of corrective, post-contingency measures is deployed to control the outputs/levels of such renewables and demand resources. As explained in [13], post-contingency, corrective control actions can successfully reduce network congestions and thus release latent network capacity of the existing assets, displacing the need for further network investments. In the case of DER, we assume that aggregators can provide three instrumental services to system operators from DER under a contingency state, namely net

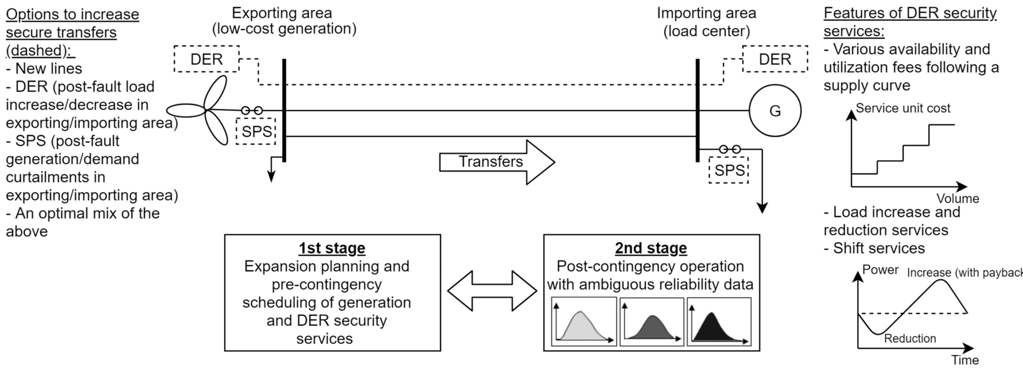

Fig. 1. Diagram illustrating the TEP problem and the general methodology.

load reduction, net load increase, and net load shifting. Although these services may be provided by a very large set of DER within a distribution network (including DG, energy storage systems and flexible load itself), they are presented/offered to transmission system operators in an aggregated fashion, using supply curves as shown in Fig. 1, where various response/volume levels (in MW) present different availability and utilization fees. In the case of the load shift service from DER, we also consider a payback effect [37], where the total energy consumption (in MWh) results, overall, higher when load is shifted.

To determine the optimal portfolio of DER security services within the TEP problem, we propose a two stage model as depicted in Fig. 1. The first stage of the model determines the transmission expansion plan and the scheduling of generation, up- and down-spinning reserves as well as the availability of DER post-contingency services. The second stage minimizes the expected cost of corrective actions under various contingencies. Since it is typically challenging to calculate precise probabilities of contingent scenarios (that is critical to determine DER support to post-contingency network congestion), in this paper, we assume limited knowledge of the underlying process behind the realization of such contingencies. Hence, we propose a formulation capable to solve the TEP problem while assuming "ambiguous" reliability data, simultaneously comprising several probability distributions of failure rates as shown in Fig. 1.

Note that as our focus is on the role of network redundancy for providing network security and how DER can efficiently compete as an alternative measure against such network redundancy, we focus on system outages rather than other sources of uncertainty in the short term. Note also that we use a static (rather than dynamic) model in the sense that it considers only one year (not many) and comprises two stages, pre and post-fault across various operating conditions. These assumptions are commonly used and well accepted in network reliability studies [8], [10], [11], [13], [14], [38].

B. Model

The two-stage TEP model with post-contingency DER services and ambiguity in the underlying process behind the realization of outages is mathematically written as follows:

$$\underset{\substack{\Delta D^+_{bte},\Delta D^-_{bte},\Delta^{FD}_{bt},\theta_{bt},\\ f_{lt},p_{it},r^d_{it},r^u_{it},u_{it},v_l}}{\text{Minimize}} \quad \sum_{t\in T} h_t \Bigg[\sum_{i\in I}\left(C^p_i p_{it} + C^d_i r^d_{it} + C^u_i r^u_{it}\right)$$
$$+ \sum_{b\in N}\Big(C^{FD}_b \Delta^{FD}_{bt} + \sum_{e\in\mathcal{E}\setminus\{\text{NE}\}}\big(C^{I+}_{be}\Delta D^+_{bte} + C^{I-}_{be}\Delta D^-_{bte}\big)\Big)$$
$$+ \sup_{\mathcal{Q}\in\mathcal{P}_t} \mathbb{E}_{\mathcal{Q}}\left\{H_t\big(p_{it}, r^d_{it}, r^u_{it}, \Delta^{FD}_{bt}, \Delta D^+_{bte}, \Delta D^-_{bte}, v_l, \boldsymbol{a}_t\big)\right\}\Bigg]$$
$$+ \sum_{l\in\mathcal{L}^C} C^L_l v_l \tag{1}$$

subject to:

$$\sum_{i\in I_b} p_{it} + \sum_{l\in\mathcal{L}|to(l)=b} f_{lt} - \sum_{l\in\mathcal{L}|fr(l)=b} f_{lt} = D_{bt};$$
$$\forall b\in N, t\in\mathcal{T} \tag{2}$$
$$-\overline{F}_l \le f_{lt} \le \overline{F}_l; \forall l\in\mathcal{L}^E, t\in\mathcal{T} \tag{3}$$
$$-v_l\overline{F}_l \le f_{lt} \le v_l\overline{F}_l; \forall l\in\mathcal{L}^C, t\in\mathcal{T} \tag{4}$$
$$f_{lt} = \frac{1}{x_l}(\theta_{fr(l),t} - \theta_{to(l),t}); \forall l\in\mathcal{L}^E, t\in\mathcal{T} \tag{5}$$
$$-M(1-v_l) + \frac{1}{x_l}(\theta_{fr(l),t} - \theta_{to(l),t}) \le f_{lt}$$
$$\le \frac{1}{x_l}(\theta_{fr(l),t} - \theta_{to(l),t}) + M(1-v_l); \forall l\in\mathcal{L}^C, t\in\mathcal{T} \tag{6}$$
$$p_{it} - r^d_{it} \ge \underline{P}_i u_{it}; \forall i\in I, t\in\mathcal{T} \tag{7}$$
$$p_{it} + r^u_{it} \le \overline{P}_i u_{it}; \forall i\in I, t\in\mathcal{T} \tag{8}$$
$$0 \le r^d_{it} \le \overline{R}^D_i; \forall i\in I, t\in\mathcal{T} \tag{9}$$
$$0 \le r^u_{it} \le \overline{R}^U_i; \forall i\in I, t\in\mathcal{T} \tag{10}$$
$$p_{wt} \le \zeta_{wt}\overline{P}_i; \forall w\in\mathcal{W}, t\in\mathcal{T} \tag{11}$$
$$0 \le \Delta^{FD}_{bt} \le \overline{\Delta}^{FD}_{bt}; \forall b\in N, t\in\mathcal{T} \tag{12}$$
$$0 \le \Delta D^+_{bte} \le \overline{\Delta D^+}_{bte}; \forall b\in N, t\in\mathcal{T}, e\in\mathcal{E}\setminus\{\text{NE}\} \tag{13}$$

$$0 \leq \Delta D^{-}_{bte} \leq \overline{\Delta D^{-}}_{bte}; \forall b \in N, t \in \mathcal{T}, e \in \mathcal{E} \setminus \{\text{NE}\} \quad (14)$$

$$v_l \in \{0,1\}; \forall l \in \mathcal{L}^C \quad (15)$$

$$u_{it} \in \{0,1\}; \forall i \in I, t \in \mathcal{T}, \quad (16)$$

where sets $\mathcal{E}$, I, I_b, $\mathcal{L}$, $\mathcal{L}^C$, $\mathcal{L}^E$, N, $\mathcal{P}_t$, $\mathcal{T}$, $\mathcal{W}$ include (in this order) indexes of steps for power imbalance costs (in this paper, steps refer to the segments – or piecewise constant values– in the supply curve shown in Fig. 1), indexes of all generators, indexes of generators connected to bus b, indexes of all transmission lines, indexes of candidate transmission lines, indexes of existing transmission lines, indexes of buses, probability distributions, indexes of time blocks (used to discretize yearly operation into a few operating points), and indexes of renewable generators such as wind and solar. The decision variables are scheduled load increase (or disconnection of DG), ΔD^{+}_{bte}, scheduled DG output increase (or load disconnection), ΔD^{-}_{bte}, scheduled load shifting, Δ^{FD}_{bt}, voltage angles, θ_{bt}, power flows, f_{lt}, power outputs, p_{it}, scheduled down- and up-spinning reserves, r^{d}_{it} and r^{u}_{it}, as well as construction of candidate lines, v_l, and commitment of dispatchable units, u_{it}. Coefficients C^{p}_{i}, C^{d}_{i}, C^{u}_{i}, C^{FD}_{b}, C^{I+}_{be}, C^{I-}_{be}, and C^{L}_{l} represent cost of generation, scheduling of down- and up-spinning reserves, scheduling of load shifting, scheduling of load increase (or disconnection of DG), scheduling of DG output increase (or load disconnection), and investment in transmission lines. Parameters $\overline{\Delta D^{+}}_{bte}$, $\overline{\Delta D^{-}}_{bte}$, $\overline{\Delta}^{FD}_{bt}$, ζ_{wt}, D_{bt}, $\overline{F}_l$, M, NE, $\underline{P}_i$, $\overline{P}_i$, $\overline{R}^{D}_{i}$, $\overline{R}^{U}_{i}$, x_l, and h_t correspond to maximum load increase (or disconnection of DG), maximum DG output increase (or load disconnection), maximum load shifting, available fraction of renewable generation at a given period, nominal demands, power flow capacities, sufficiently large constant, number of steps for power imbalance cost, minimum stable generations, maximum stable generations, maximum capacities of down-spinning reserve, maximum capacities of up-spinning reserve, reactances of lines, and number of hours in each time block, respectively. Finally, $H_t(\boldsymbol{q}_t, \boldsymbol{a}_t)$ represents generation cost for a given first-stage decision, $\boldsymbol{q}_t$, and $\boldsymbol{a}_t$ is a random vector associated with the availability of system elements (generators and transmission lines), i.e, $\boldsymbol{a}_t = (\boldsymbol{a}^{G^T}_{t}, \boldsymbol{a}^{L^T}_{t})^T$.

The objective function (1) to be minimized includes costs of generation and scheduling of post-contingency services, namely, down- and up-spinning reserves, load shifting, load increase (or disconnection of DG), and DG output increase (or load disconnection) as well as expected value of second-stage operation cost, and cost of construction of candidate lines. Nodal power balance is modeled by constraints (2). Constraints (3) and (4) enforce power flow limits for existing and candidate lines, respectively. In a DC load flow fashion, constraints (5) and (6) represent power transfers through existing and candidate lines, respectively. Constraints (7)–(10) impose limits to pre-contingency power generation as well as down- and up-spinning reserves scheduling for each dispatchable unit. Similarly, constraints (11) limit the generation of renewable generation units. Constraints (12)–(14) model maximum capacities of post-contingency DER services. It should be noted that the last step e of variables $\Delta D^{+}{}_{bte}$ and $\Delta D^{-}{}_{bte}$ is not scheduled in the first stage since such step corresponds to generation and demand curtailments (i.e. SPS), which are highly penalized in the second stage. Finally, constraints (15) and (16) enforce the binary nature of investment and commitment variables.

The operation under contingency can be modeled as:

$$H_t(\boldsymbol{q}_t, \boldsymbol{a}_t) = \underset{\substack{\Delta^{FD+c}_{bty}, \Delta^{FD-c}_{bty}, \\ \Delta D^{+c}_{btye}, \Delta D^{-c}_{btye}, \\ \theta^{c}_{bty}, f^{c}_{lty}, p^{c}_{ity}, \\ r^{dc}_{ity}, r^{uc}_{ity}}}{\text{Minimize}} \sum_{y \in \mathcal{Y}} \Big[\sum_{i \in I} \left(C^{dc}_{i} r^{dc}_{ity} + C^{uc}_{i} r^{uc}_{ity} \right) \Delta t$$
$$+ \sum_{b \in N, e \in \mathcal{E}} \left(C^{I+^c}_{be} \Delta D^{+c}_{btye} + C^{I-^c}_{be} \Delta D^{-c}_{btye} \right) \Delta t \Big]$$
$$+ \sum_{b \in N} C^{FD^c}_{b} \Delta^{FD-c}_{bt1} \quad (17)$$

subject to:

$$\sum_{i \in I_b} p^{c}_{ity} + \sum_{l \in \mathcal{L} | to(l) = b} f^{c}_{lty} - \sum_{l \in \mathcal{L} | fr(l) = b} f^{c}_{lty}$$
$$+ \sum_{e \in \mathcal{E}} \left(\Delta D^{-c}_{btye} - \Delta D^{+c}_{btye} \right) + \Delta^{FD-c}_{bty} - \Delta^{FD+c}_{bty} = D_{bt} :$$
$$(\lambda_{bty}); \forall b \in N, \, y \in \mathcal{Y} \quad (18)$$

$$-a^{L}_{lt} \overline{F}_l \leq f^{c}_{lty} \leq a^{L}_{lt} \overline{F}_l : (\phi^{+}_{lty}, \phi^{-}_{lty}); \forall l \in \mathcal{L}^E, \, y \in \mathcal{Y} \quad (19)$$

$$-a^{L}_{lt} v_l \overline{F}_l \leq f^{c}_{lty} \leq a^{L}_{lt} v_l \overline{F}_l : (\phi^{n+}_{lty}, \phi^{n-}_{lty}); \forall l \in \mathcal{L}^C,$$
$$y \in \mathcal{Y} \quad (20)$$

$$-M(1 - a^{L}_{lt}) + \frac{1}{x_l} (\theta^{c}_{fr(l),ty} - \theta^{c}_{to(l),ty}) \leq f^{c}_{lty}$$
$$\leq \frac{1}{x_l} (\theta^{c}_{fr(l),ty} - \theta^{c}_{to(l),ty}) + M(1 - a^{L}_{lt}) : (\mu^{+}_{lty}, \mu^{-}_{lty});$$
$$\forall l \in \mathcal{L}^E, y \in \mathcal{Y} \quad (21)$$

$$-M(1 - v_l a^{L}_{lt}) + \frac{1}{x_l} (\theta^{c}_{fr(l),ty} - \theta^{c}_{to(l),ty}) \leq f^{c}_{lty}$$
$$\leq \frac{1}{x_l} (\theta^{c}_{fr(l),ty} - \theta^{c}_{to(l),ty}) + M(1 - v_l a^{L}_{lt}) : (\mu^{n+}_{lty}, \mu^{n-}_{lty});$$
$$\forall l \in \mathcal{L}^C, y \in \mathcal{Y} \quad (22)$$

$$p^{c}_{ity} = p_{it} a^{G}_{it} + r^{uc}_{ity} - r^{dc}_{ity} : (\eta_{ity}); \forall i \in I, y \in \mathcal{Y} \quad (23)$$

$$0 \leq r^{dc}_{ity} \leq r^{d}_{it} a^{G}_{it} : (\kappa^{-}_{ity}); \forall i \in I, y \in \mathcal{Y} \quad (24)$$

$$0 \leq r^{uc}_{ity} \leq r^{u}_{it} a^{G}_{it} : (\kappa^{+}_{ity}); \forall i \in I, y \in \mathcal{Y} \quad (25)$$

$$p^{c}_{it1} \leq \left(p_{it} + RU_i \frac{\Delta t}{2} \right) a^{G}_{it} : (\chi^{+}_{it}); \forall i \in I \quad (26)$$

$$p^{c}_{it1} \geq \left(p_{it} - RD_i \frac{\Delta t}{2} \right) a^{G}_{it} : (\chi^{-}_{it}); \forall i \in I \quad (27)$$

$$p^{c}_{ity} - p^{c}_{ity-1} \leq RU_i \Delta t : (o^{+}_{ity}); \forall i \in I, y \in \mathcal{Y} \setminus \{1\} \quad (28)$$

$$p^c_{ity-1} - p^c_{ity} \leq RD_i \Delta t : (o^-_{ity}); \forall i \in I, y \in \mathcal{Y} \setminus \{1\} \quad (29)$$

$$0 \leq \Delta D^{+c}_{btye} \leq \Delta D^{+}_{bte} : (\psi^+_{btye}); \forall b \in N, y \in \mathcal{Y}, e \in \mathcal{E} \setminus \{\text{NE}\} \quad (30)$$

$$0 \leq \Delta D^{-c}_{btye} \leq \Delta D^{-}_{bte} : (\psi^-_{btye}); \forall b \in N, y \in \mathcal{Y}, e \in \mathcal{E} \setminus \{\text{NE}\} \quad (31)$$

$$0 \leq \Delta^{FD-c}_{bty} \leq \Delta^{FD}_{bt} : (\rho_{bty}); \forall b \in N, y \in \mathcal{Y} \quad (32)$$

$$\delta \sum_{y \in \mathcal{Y}} \Delta^{FD-c}_{bty} = \sum_{y \in \mathcal{Y}} \Delta^{FD+c}_{bty} : (\iota_{bt}); \forall b \in N \quad (33)$$

$$\Delta^{FD-c}_{bt1} \geq \Delta^{FD-c}_{bty} : (\beta_{bty}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\} \quad (34)$$

$$\Delta D^{+c}_{bt1e} \geq \Delta D^{+c}_{btye} : (\gamma^+_{btye}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\}, e \in \mathcal{E} \setminus \{\text{NE}\} \quad (35)$$

$$\Delta D^{-c}_{bt1e} \geq \Delta D^{-c}_{btye} : (\gamma^-_{btye}); \forall b \in N, y \in \mathcal{Y} \setminus \{1\}, e \in \mathcal{E} \setminus \{\text{NE}\} \quad (36)$$

$$\Delta D^{+c}_{b,t,y,NE} \leq D_{bt} : (\sigma^+_{bty}); \forall b \in N, y \in \mathcal{Y} \quad (37)$$

$$\Delta D^{-c}_{b,t,y,NE} \leq \sum_{i \in I_b} p_{it} a^G_{it} : (\sigma^-_{bty}); \forall b \in N, y \in \mathcal{Y}, \quad (38)$$

where $\mathcal{Y}$ is the set of indexes of snapshots under contingency within each time block $t \in \mathcal{T}$. These snapshots are used to discretize post-contingency operation and capture evolution of relevant variables within 1 hour. For instance, if set $\mathcal{Y}$ comprises two snapshots, the first one is related to the first 30 minutes after the occurrence of an outage and the second one corresponds to the remaining 30 minutes. Parameters Δt, δ, RD_i, and RU_i represent time length of each snapshot (1 hour divided by the number of snapshots), payback for the load shifting service, ramp-down and ramp-up limit of each generator, respectively. Coefficients C^{dc}_i, C^{uc}_i, $C^{I+^c}_{be}$, $C^{I-^c}_{be}$, and $C^{FD^c}_b$ correspond to costs of utilizing scheduled down-spinning reserve, up-spinning reserve, DG disconnection (or load increase), load decrease (or DG output increase), and load shifting, respectively. The decision variables correspond to the positive and negative deviation of flexible demand from its nominal value, Δ^{FD+c}_{bty} and Δ^{FD-c}_{bty}, actual DG disconnection (or load increase), ΔD^{+c}_{btye}, actual load decrease (or DG output increase) ΔD^{-c}_{btye}, voltage angles under contingency, θ^c_{bty}, power transfers under contingency, f^c_{lty}, generation output under contingency, p^c_{ity}, utilized down- and up-spinning reserves under contingency, r^{dc}_{ity} and r^{uc}_{ity}.

The objective function (17) to be minimized in the system operation problem includes costs of utilizing scheduled post-contingency services of down- and up-spinning reserves, DG disconnection (or load increase), load decrease (or DG output increase), and load shifting. Analogously to (2)–(6), constraints (18)–(22) model nodal balance and power transfers under contingency. Constraints (23) relate post-contingency generation outputs to pre-contingency generation and scheduled reserves. Constraints (24) and (25) limit the utilization of down- and up-spinning reserves to the amounts scheduled in the first-stage. Constraints (26)–(29) impose ramp rate limits to the post-contingency generation. Note that constraints (26) and (27) (which are imposed only on the first snapshot) present a term equal to $\Delta t/2$ since this is the middle or reference point in time to which the variables of the first snapshot are referred. In (28) and (29), we do not divide Δt by 2 as the time length between the middle/reference points of two consecutive snapshots is equal to Δt. Constraints (30) and (31) enforce limits for actual DG disconnection (or load increase) and actual load decrease (or DG output increase), respectively, for all steps of power imbalance costs, except the last one which corresponds to involuntary generation curtailment $\left(\Delta D^{+c}_{b,t,y,NE}\right)$ or demand curtailment $\left(\Delta D^{-c}_{b,t,y,NE}\right)$. Constraints (32) and (33) model actual load shifting limits according to first-stage decision and load shifting payback, respectively. Constraints (34)–(36) impose as a rule that the first snapshot ($y = 1$) should present the highest values of load shifting as well as DG disconnection (or load increase) and load decrease (or DG output increase). We use this rule because the first snapshot represents the first minutes right after an outage occurs and therefore when the volume of corrective control measures is the highest. Finally, (37) and (38) impose limits on involuntary generation curtailment $\left(\Delta D^{+c}_{b,t,y,NE}\right)$ and demand curtailment $\left(\Delta D^{-c}_{b,t,y,NE}\right)$, respectively.

Note that, due to our focus on network security, ramp rate limits (and other operational details) have been ignored in the first stage but considered in the second stage in order to properly compare DER against utilization of reserve services right after an outage occurs. Interestingly, considering the lack of generation flexibility in the first stage (ignored in this paper) can enhance the scope of the benefits associated with DER (which are associated with further ancillary services, apart from network security), but this is beyond the scope of this paper.

C. Ambiguity Sets

Similar to [30], for each time block $t \in \mathcal{T}$, we consider the ambiguity set $\mathcal{P}_t$ which is constituted by the probability distributions associated with the $n - K$ criterion for a given knowledge level of failure probabilities. The ambiguity set $\mathcal{P}_t$ can be mathematically described as:

$$\mathcal{P}_t = \left\{ \mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \mathbb{E}_{\mathcal{Q}}[S \hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t \right\}, \quad (39)$$

where $\hat{\boldsymbol{a}}_t = \mathbb{1} - \boldsymbol{a}_t$. Vector $\overline{\boldsymbol{\mu}}_t$ and matrix S correspond to estimated values of means and an auxiliary matrix of coefficients.

In addition, we define:

$$\mathcal{A} = \Big\{ (\boldsymbol{a^G}, \boldsymbol{a^L}) \in \{0,1\}^{|I|} \times \{0,1\}^{|\mathcal{L}|} : \sum_{i \in I} a^G_i + \sum_{l \in \mathcal{L}} a^L_l \geq n - K \Big\}, \quad (40)$$

where n is the number of system elements ($n = |I| + |\mathcal{L}|$) and K is the security parameter, which is a predefined number of simultaneous outages.

In the presented methodology, $\boldsymbol{a}_t$ is a random vector that represents the availability of system elements (in our case, generators and transmission lines). In addition, set $\mathcal{A}$ is the support of $\boldsymbol{a}_t$. Within this context, $\mathcal{M}_+(\mathcal{A})$ contains all probability distributions on $\mathcal{A}$. Thus, $\mathcal{Q}$ is a probability distribution that belongs to $\mathcal{M}_+(\mathcal{A})$ such that the condition $\mathbb{E}_\mathcal{Q}[S\hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t$ is satisfied. Hence, for each time block t, $\mathcal{P}_t$ contains all probability distributions in $\mathcal{M}_+(\mathcal{A})$ that comply with $\mathbb{E}_\mathcal{Q}[S\hat{\boldsymbol{a}}_t] \leq \overline{\boldsymbol{\mu}}_t$.

In this paper, within the context of transmission expansion planning, we compare three types of ambiguity sets, namely fixed probabilities, $n - K$ security, and interval probabilities, which are described next.

1) Fixed Probabilities Approach: In this case, we consider that failure probabilities are well-known, therefore, we have:

$$\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \mathbb{E}_\mathcal{Q}[1 - a_{it}^G] = p_{it}^G, \forall i \in I; \\ \mathbb{E}_\mathcal{Q}[1 - a_{lt}^L] = p_{lt}^L, \forall l \in \mathcal{L}\big\}; \forall t \in \mathcal{T}. \quad (41)$$

For this approach $\overline{\boldsymbol{\mu}}_t$ should be chosen as $[p^T, -p^T]^T$, where p is the column vector of estimated failure rates. Matrix S would be $[\mathbb{I}, -\mathbb{I}]^T \in \mathbb{R}^{2(|I|+|L|)\times(|I|+|L|)}$.

2) $n - K$ Security Approach: Opposite to the previous approach, in this case, we assume that failure probabilities are completely unknown. Hence, the ambiguity set is defined as:

$$\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : 0 \leq \mathbb{E}_\mathcal{Q}[1 - a_{it}^G] \leq 1, \forall i \in I; \\ 0 \leq \mathbb{E}_\mathcal{Q}[1 - a_{lt}^L] \leq 1, \forall l \in \mathcal{L}\big\}; \forall t \in \mathcal{T}. \quad (42)$$

In this case, $\overline{\boldsymbol{\mu}}_t$ should be chosen as $[\vec{1}^T, \vec{0}^T]^T$, where $\vec{1}$ and $\vec{0}$ are column vectors of only ones and zeros in $\mathbb{R}^{|I|+|L|}$, respectively. Whereas matrix S is the same used for the previous case.

3) Interval Probabilities Approach: In this approach, we assume limited knowledge of the underlying process behind the realization of outages. This knowledge is characterized by a range of failure probabilities (i.e. ambiguity intervals), whose length depends on the quality of the historical data regarding outages of the element (more details on the definition of distributional sets under moment uncertainty can be found in [27]), and a bound of the overall system's failure rate. In this manner, the overall ambiguity set is written as:

$$\mathcal{P}_t = \big\{\mathcal{Q} \in \mathcal{M}_+(\mathcal{A}) : \underline{p}_{it}^G \leq \mathbb{E}_\mathcal{Q}[1 - a_{it}^G] \leq \overline{p}_{it}^G, \forall i \in I; \\ \underline{p}_{lt}^L \leq \mathbb{E}_\mathcal{Q}[1 - a_{lt}^L] \leq \overline{p}_{lt}^L, \forall l \in \mathcal{L}; \sum_{i\in I} \mathbb{E}_\mathcal{Q}[1 - a_{it}^G] \\ + \sum_{l\in\mathcal{L}} \mathbb{E}_\mathcal{Q}[1 - a_{lt}^L] \leq p_t\big\}; \forall t \in \mathcal{T}. \quad (43)$$

The vector of estimated means, $\overline{\boldsymbol{\mu}}_t$, should be selected as $[\overline{p}^T, -\underline{p}^T, p]^T$ for this case. Where $\overline{p}$ is the column vector of upper bounds for the failure rates, $\underline{p}$ is the column vector for lower bounds, and p is the system wide failure rate. This time matrix S should be chosen as $[\mathbb{I}, -\mathbb{I}, \vec{1}]^T \in \mathbb{R}^{(2(|I|+|L|)+1)\times(|I|+|L|)}$.

III. Solution Methodology

The two-stage model (1)–(16) mathematically describes the TEP problem with an optimal portfolio of DER security services under ambiguity in failure probabilities of system equipments. Due to convenient convexity properties, this model is suitable for the use of Benders decomposition.

First, for simplicity purposes, the two-stage model (1)–(16) can be written in the following compact manner.

$$\underset{\boldsymbol{q}_t}{\text{Minimize}} \quad \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\Big] \\ + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \sum_{t\in T} h_t \sup_{\mathcal{Q}\in\mathcal{P}_t} \mathbb{E}_\mathcal{Q}(H_t(\boldsymbol{q}_t, \boldsymbol{a}_t)) \quad (44)$$

subject to:

$$A\boldsymbol{q}_t \geq \boldsymbol{b}_t; \forall t \in \mathcal{T} \quad (45)$$

$$\boldsymbol{q}^L \in \{0,1\}^{|\mathcal{L}^C|} \quad (46)$$

$$\boldsymbol{q}_t^B \in \{0,1\}^{|I||T|}, \quad (47)$$

where $\boldsymbol{q}_t^B$, $\boldsymbol{q}_t^C$, and $\boldsymbol{q}^L$ represent vectors of binary operational variables, continuous operational variables, and binary investment variables, respectively. Note that $\boldsymbol{q}_t = [\boldsymbol{q}_t^{B^T}, \boldsymbol{q}_t^{C^T}, \boldsymbol{q}^{L^T}]^T$. In addition, the objective function (44) corresponds to (1), whereas constraints (45) group (2)–(14). Moreover (46) and (47) are related to (15) and (16), respectively.

Also, the operation under uncertainty (17)–(38) can be written as:

$$H_t(\boldsymbol{q}_t, \boldsymbol{a}_t) = \underset{\boldsymbol{y}_t}{\text{Minimize}} \quad d_t^T \boldsymbol{y}_t \quad (48)$$

subject to:

$$B_t\boldsymbol{y}_t \geq e_t : (\Theta_t) \quad (49)$$

$$C_t\boldsymbol{y}_t \geq D_t\boldsymbol{q}_t + g_t : (\Phi_t) \quad (50)$$

$$E_t\boldsymbol{y}_t \geq F_t(\boldsymbol{a}_t)\boldsymbol{q}_t + h_t(\boldsymbol{a}_t) : (\Omega_t) \quad (51)$$

$$G_t\boldsymbol{y}_t \geq J_t(\boldsymbol{a}_t)\boldsymbol{q}_t + j_t : (\Lambda_t) \quad (52)$$

$$K_t\boldsymbol{y}_t \geq s_t(\boldsymbol{a}_t) : (\Gamma_t), \quad (53)$$

where (48) represents (17). Expression (49) groups constraints (18), (28), (29), (33)–(37). Constraint (50) is associated with (30)–(32), whereas (51) is related to (26) and (27). Expression (52) represents (20), (22)–(25), and (38). Finally, constraint (53) corresponds to (19) and (21).

The steps related to the proposed solution methodology are described next.

A. Problem Reformulation

Formulation (44)–(47) can be equivalently written as the following bilevel program:

$$\underset{\alpha_t, \boldsymbol{q}_t}{\text{Minimize}} \quad \sum_{t\in T} h_t \Big[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\Big] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L \\ + \sum_{t\in T} h_t\alpha_t \quad (54)$$

subject to:

Constraints (45)–(47) (55)

$$\alpha_t = \Bigg\{ \underset{\mathcal{Q}\in\mathcal{P}_t}{\text{Maximize}} \quad \sum_{a_t\in\mathcal{A}} H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big)\mathcal{Q}(\boldsymbol{a}_t) \quad (56)$$

subject to:

$$\sum_{a_t \in \mathcal{A}} S\hat{\boldsymbol{a}}_t \mathcal{Q}(\boldsymbol{a}_t) \leq \overline{\boldsymbol{\mu}}_t : (\boldsymbol{\pi}_t) \tag{57}$$

$$\left.\sum_{a_t \in \mathcal{A}} \mathcal{Q}(\boldsymbol{a}_t) = 1 : (\varphi_t)\right\}, \forall t \in \mathcal{T}, \tag{58}$$

where the adequate choice of matrix S and vector $\overline{\boldsymbol{\mu}}_t$ in expression (57) indicate which of the ambiguity sets defined in (41), (42), and (43) is considered.

In light of duality theory, model (54)–(58) can be rewritten as:

$$\underset{\boldsymbol{\pi}_t \geq 0, \varphi_t, \boldsymbol{q}_t}{\text{Minimize}} \quad \sum_{t \in T} h_t \left[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\right] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \sum_{t \in T} h_t \left(\boldsymbol{\pi}_t^T \overline{\boldsymbol{\mu}}_t + \varphi_t\right) \tag{59}$$

subject to:

$$\text{Constraints (45)–(47)} \tag{60}$$

$$\boldsymbol{\pi}_t^T S\hat{\boldsymbol{a}}_t + \varphi_t \geq H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big), \forall t \in \mathcal{T}, \boldsymbol{a}_t \in \mathcal{A}. \tag{61}$$

It should be noted that constraints (61) can render formulation (59)–(61) computationally intractable due to their combinatorial nature. In order to circumvent this dimensionality curse, we rewrite (61) in the following manner.

$$\varphi_t \geq \max_{\boldsymbol{a}_t \in \mathcal{A}} \Big\{ H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big) - \boldsymbol{\pi}_t^T S\hat{\boldsymbol{a}}_t \Big\}, \forall t \in \mathcal{T}. \tag{62}$$

Based on the presented reformulation, we describe next subproblem, master problem, and solution algorithm.

B. Subproblem

At each iteration j, for a given first-stage solution $\boldsymbol{q}_t^{(j)}$, the subproblem identifies its associated worst-case contingency, $\boldsymbol{a}_t$, for each time block $t \in \mathcal{T}$. This identification is done by means of the problem formulated in the right hand side of (62). This problem is bilevel program since $H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big)$ corresponds to a minimization problem and it is therefore not aligned with the outer maximization in $\boldsymbol{a}_t \in \mathcal{A}$. However, since $H_t\big(\boldsymbol{q}_t, \boldsymbol{a}_t\big)$ is a linear program, we replace it by its dual (which is a maximization problem) in (62), include constraints (66)–(68), and linearize products of binary and continuous decision variables to develop a mixed integer linear programming (MILP) formulation hereinafter called subproblem. Hence, the subproblem can be represented by the compact formulation (63)–(68). The complete formulation of the subproblem can be found in [39].

$$\underset{\Theta_t, \Phi, \Omega_t, \Lambda_t, \Gamma_t, \boldsymbol{a}_t}{\text{Maximize}} \quad \sum_{t \in T} \big[e_t^T \Theta_t + (g_t + D_t \boldsymbol{q}_t)^T \Phi_t + (F_t(\boldsymbol{a}_t)\boldsymbol{q}_t + h_t(\boldsymbol{a}_t))^T \Omega_t + (J_t(\boldsymbol{a}_t)\boldsymbol{q}_t + j_t)^T \Lambda_t + s_t(\boldsymbol{a}_t)^T \Gamma_t - \boldsymbol{\pi}_t^T S\hat{\boldsymbol{a}}_t\big] \tag{63}$$

subject to:

$$B_t^T \Theta_t + C_t^T \Phi_t + E_t^T \Omega_t + G_t^T \Lambda_t + K_t^T \Gamma_t = d_t; \forall t \in T \tag{64}$$

$$\Theta_t, \Phi_t, \Omega_t, \Lambda_t, \Gamma_t \geq 0; \forall t \in T \tag{65}$$

$$\sum_{l \in \mathcal{L}} a_{lt}^L + \sum_{i \in i} a_{it}^G \geq n - K; \forall t \in T \tag{66}$$

$$a_{lt}^L \in \{0,1\}; \forall l \in \mathcal{L}, t \in T \tag{67}$$

$$a_{it}^G \in \{0,1\}; \forall i \in I, t \in T \tag{68}$$

C. Master Problem

The master problem (69)–(71) is a relaxation of the original problem (1)–(16). This relaxation is achieved by replacing (62) by cutting planes in the equivalent reformulation of the original problem (59)–(60) and (62). The complete formulation of the master problem can be found in [39].

$$\underset{\boldsymbol{\pi}_t \geq 0, \varphi_t, \boldsymbol{q}_t}{\text{Minimize}} \quad \sum_{t \in T} h_t \left[\boldsymbol{c}^T \boldsymbol{q}_t^C + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^B\right] + \boldsymbol{c}^{L^T} \boldsymbol{q}^L + \sum_{t \in T} h_t \left(\boldsymbol{\pi}_t^T \overline{\boldsymbol{\mu}}_t + \varphi_t\right) \tag{69}$$

subject to:

$$\text{Constraints (45)–(47)} \tag{70}$$

$$\varphi_t \geq e_t^T \Theta_t^{(j)} + (g_t + D_t \boldsymbol{q}_t)^T \Phi_t^{(j)} + (F_t(\boldsymbol{a}_t^{(j)})\boldsymbol{q}_t + h_t(\boldsymbol{a}_t^{(j)}))^T \Omega_t^{(j)} + (J_t(\boldsymbol{a}_t^{(j)})\boldsymbol{q}_t + j_t)^T \Lambda_t^{(j)} + s_t(\boldsymbol{a}_t^{(j)})^T \Gamma_t^{(j)} - \boldsymbol{\pi}_t^T S\hat{\boldsymbol{a}}_t^{(j)}; \forall j \in \mathcal{J}, t \in T \tag{71}$$

D. Solution Algorithm

The procedure proposed in this section is an outer algorithm based on Benders decomposition. This outer algorithm is an iterative process that is carried out until the included cutting planes rend the solution of the relaxed problem (master problem) sufficiently close to optimality. The solution algorithm can be summarized as follows.

1) Initialization: set $j \leftarrow 0$.
2) Solve the optimization model (69)–(71), store $\boldsymbol{q}_t^{(j)}$, $\boldsymbol{\pi}_t^{(j)}$ and $\varphi_t^{(j)}$, and calculate $LB^{(j)} = \sum_{t \in T} h_t[\boldsymbol{c}^T \boldsymbol{q}_t^{C(j)} + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^{B(j)}] + \boldsymbol{c}^{L^T} \boldsymbol{q}^{L(j)} + \sum_{t \in T} h_t(\boldsymbol{\pi}_t^{(j)T} \overline{\boldsymbol{\mu}}_t + \varphi_t^{(j)})$.
3) Identify the worst case contingency for stored $\boldsymbol{q}_t^{(j)}$ and $\boldsymbol{\pi}_t^{(j)}$ by running the subproblem, store values of its decision variables and calculate $UB^{(j)} = \sum_{t \in T} h_t[\boldsymbol{c}^T \boldsymbol{q}_t^{C(j)} + \boldsymbol{c}^{B^T} \boldsymbol{q}_t^{B(j)}] + \boldsymbol{c}^{L^T} \boldsymbol{q}^{L(j)} + \sum_{t \in T} h_t(\boldsymbol{\pi}_t^{(j)T} \overline{\boldsymbol{\mu}}_t + \Psi_t^{(j)})$, where $\Psi_t^{(j)}$ is the value of the objective function of the subproblem for time block t.
4) If $\big(UB^{(j)} - LB^{(j)}\big)/UB^{(j)} \leq \epsilon$, then STOP ; else, CONTINUE.
5) Include in (69)–(71) a cutting plane of the format (71) with decision variables stored in step 3, set $j \leftarrow j + 1$, and go to step 2.

IV. IEEE RTS Case Study

This section studies the economic and reliability performance of various security services provided by DER when co-optimized with network investments and other alternative operational measures to release network capacity. To do so, we introduce three approaches for the treatment of DER security services, namely improved $n-1$ security, fixed probabilities and interval probabilities. These three approaches will be compared against the traditional $n-1$ security approach that prevents the use of DER services to provide network security.

A. Input Data

We modified the IEEE RTS described in [40] by adding:

1) 500 MW of thermal generation in buses 13, 14, 16, 23.
2) 700 MW of peak load in buses 9,10, and 400 MW of peak load in bus 3.
3) 300 MW of wind generation in bus 12.

Also, we consider the following 10 candidate network assets (in addition to the existing infrastructure informed in [40]) in the network planning problem (indicating end buses): 3–24, 9–11, 9–12, 10–11, 10–12, 11–13, 11–14, 12–13, 12–23, 15–24. Network investment costs for lines and transformers are 60 \$/MW.km.year and 20 k\$/MW.year, respectively. The annuity of the investment cost is balanced against the cost of one year of system operation, representing the state of the system when the transmission assets are already built, that is, years after the investment decisions have been originally made. Other relevant cost data includes a VoLL equal to 12 k\$/MWh, SPS utilization cost for generation curtailment equal to 1 k\$/MWh, and reserve utilization costs equal to the fuel costs of the corresponding generation technologies (fuel costs can be found in [41]). Cost of holding/scheduling generation reserves is considered to be costlier for fast generation technologies and equal to 20 \$/MW/h. For slow generation technologies, we consider a lower cost equal to 7 \$/MW/h. Regarding the pre-contingency operating conditions, they were clustered in 40 blocks that represent combinations of different demand and wind outputs across a year. Regarding post-fault operation, all $n-1$ contingencies are modeled throughout 1 hour, divided into 2 30-min snapshots. Outage rates of network and generation equipment are those presented in [40].

We consider DER post-contingency services available in 10 nodes with the following features:

1) Downwards DER service: Disconnections of flexible, non-essential loads and DG outputs increases that can contribute up to 13% of the nodal demand and respond right after a contingency occurs. Scheduling costs follow a 2-step supply curve similar to that illustrated in Fig. 1 whose values are equal to 5 and 10 \$/MW/h for the first 8% and the following 5%, respectively. Likewise, utilization costs are equal to 50 and 80 \$/MWh.
2) Upwards DER service: Disconnections of DG and demand increases that can contribute up to an equivalent of 6% of nodal demand and respond right after a contingency occurs. Scheduling costs follow a 2-step supply curve similar to that illustrated in Fig. 1 whose values are equal to 1 and 2 \$/MW/h for the first 4% and the following 2%, respectively. Likewise, utilization costs are equal to 20 and 30 \$/MWh.
3) Shift DER service: Shifts of flexible, non-essential loads and storage plants that can contribute up to 5% of nodal demand, responding right after a contingency occurs and recovering 30 min later. Scheduling and utilization costs are equal to 2 \$/MW/h and 5 \$/MW, respectively. The payback considers a 10% penalization in terms of energy consumption (parameter $\delta = 1.1$ in (33)).

B. Case Studies

We analyze 4 approaches to consider DER security services in network investment planning:

1) Traditional $n-1$ security approach (baseline), where no DER is permitted.
2) Improved $n-1$ security approach, where DER services are used to provide security as long as involuntary demand curtailments through SPS are avoided. Post-contingency costs are, evidently, neglected since probabilities are ignored.
3) Fixed probabilities, where DER services are used to provide security in coordination with further post-contingency control measures (i.e. SPS).
4) Interval probabilities, where DER services are used to provide security in coordination with further post-contingency control measures, recognizing ambiguity in reliability data.

We use Julia version 0.6 and Gurobi [42] on a server with two 10-core processors (Intel Xeon E5-2660) and 48 GB of RAM.

C. Results and Discussion

Table I presents a general overview of the results for each approach, where economic, reliability and physical output data are shown and compared. Discussion is provided next.

1) Case 1: Traditional $n-1$ Solution (baseline, no DER): Table I shows that, in the baseline case, the model invests significantly, including 7 network assets (3 lines and 4 transformers) at a total investment cost of \$38.3 million. Also, the amount of reserve scheduled is the highest and equal to 398 MW (on average across the year) in order to deal with all $n-1$ generation (and network) outages without demand participation, totalizing an annual scheduling cost equal to \$60.6 million. Given that DER is prevented to provide security services, high levels of redundancy in both transmission and generation are needed to secure the system.

2) Case 2: Improved $n-1$ Solution With DER Participation: In this case, Table I shows that the model invests in 5 network assets (2 lines and 3 transformers), totalizing an investment cost of \$27.1 million, which is considerably lower than that of the traditional $n-1$ approach. Furthermore, the model also schedules considerably less generation reserves, with a total reserve scheduling cost of \$19.1 million per year and an average of 155 MW of generation reserve margin across the year. Expectedly, this is possible due to the scheduling of DER services, which totalizes a cost equal to \$15.7 million. All of the above demonstrate, at least from a robust/deterministic point of view, the significant benefits of DER.

TABLE I
OVERALL RESULTS OF 4 DER APPROACHES

Item	Traditional n-1 (baseline, no DER)	Improved n-1	Fixed probabilities	Interval probabilities
Investment cost [million $]	38.3	27.1	17.7	19.1
Operating cost (planned, pre-fault) [million $]	295.7	289.3	283.7	283.5
Reserve holding cost (all services) [million $]	60.6	19.1	12.5	12.7
DER holding cost (all services) [million $]	0	15.7	15.3	15.3
Expected costs of generation reserves DER SPS [million $]*	0.46 0 0.75	0.31 0.59 0.61	0.22 0.66 4.00	0.22 0.64 2.75
Total cost [million $]	395.8	352.71	334.1	334.2
$\text{CVaR}_{99\%}$ of SPS cost [million $]*	75.5	60.7	398.4	273.9
LOLE [h]*	0.48	0.40	7.67	4.06
No. of new network assets installed	7	5	3	4
Averaged upwards/ downwards generation reserve available [MW]	398/99	155/92	150/0	152/0
Averaged downwards/ upwards DER services available [MW]	0/0	250/29	250/9	250/17
Averaged shift DER service available [MW]	0	10	7	8

*Results obtained from an out-of-sample analysis of 30 million scenarios (3,000 random vectors where each vector element contains the failure rate of a system component uniformly distributed in the $[-30\%, 30\%]$ ambiguity interval with respect to its reference value, times 10,000 contingencies (beyond $n-1$) for each of these vectors, considering exponentially distributed outages as indicated in [43]).

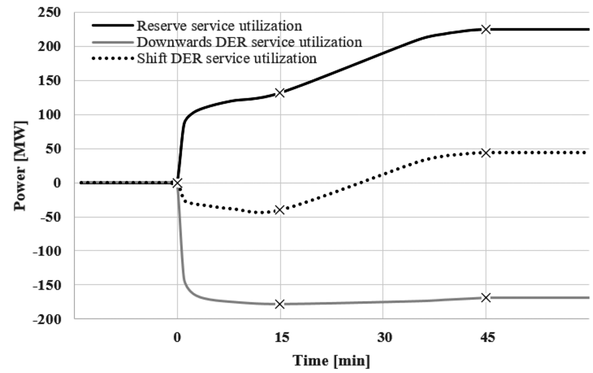

Fig. 2. Aggregated utilization of reserves and DER services when facing an outage of a 350 MW generator on bus 23. Model results are indicated by crosses; for the sake of visualization, referential lines are provided.

3) Case 3: Fixed Probabilities Approach Solution With DER Participation: In this case, the optimal portfolio of DER and further post-contingency control actions can significantly displace redundancy (network capacity and generation reserves), reducing network investment and generation reserve availability costs up to \$17.7 and 12.5 million, respectively.

Regarding the more efficient use of generation reserves, we observe an interesting interaction with DER services. In particular, we notice that optimally shifting non-essential loads allows operators to delay (rather than reduce) the need for generation reserve utilization. This enables the use of slower, less flexible thermal units (but more cost-effectively) to provide reserves services. Fig. 2 shows exercised volumes of reserves and DER services, demonstrating that faster (and more costly) generation reserves can be reduced and interchanged by slower (and lower-cost) generation reserves due to the use of shift DER services, improving the overall economic performance of post-contingency control actions to secure the power network.

4) Case 4: Interval Probabilities Approach Solution With DER Participation: We run the proposed interval probabilities approach to DER, where each failure rate is assumed to be within the $\pm 30\%$ ambiguity interval with respect to its reference value. Also, we assume (following [30]) that the aggregated system failure rate, i.e. overall number of outages at a system level, presents an upper bound and equal to that of the previous fixed probabilities approach (see Eq. (43)). This assumption properly captures that, at an aggregated level, uncertainty is more limited.

Because of the ambiguity in reliability data, the model hedges the system operation and investment solutions through higher volumes of network investment and generation reserves as shown in Table I in comparison with the fixed probabilities model. Furthermore, we found that the interval probabilities approach solution seeks to protect the system against the outages of highly loaded generating units and lines that, due to the ambiguity framework, become more prone to fail due to their potentially large impact. Similarly to the $n-1$ criterion that protects the system against the worst $n-1$ outage, the interval probabilities approach protects the system against the worst expected impact, that with the largest risk (i.e. worst expected cost; note that worst cases are decision dependent). In this context, this model decides to reinforce transmission corridors from/to busbar 11, which, after examining the resulting network operation, is proved to be the main hub of the system (the one with the highest volumes of energy transferred across a year).

D. Overall Costs and Risks: Out-of-Sample Analysis

Table I shows material benefits of DER on total costs and risks, although the ultimate value of DER depends on the approach used towards probabilities. In fact, Table I shows that ignoring reliability data (like in the improved $n-1$ approach) significantly undermines the value of DER services. Furthermore, Table I demonstrates that the fixed probabilities approach can minimize total costs mainly through less network investment. Similarly, the interval probabilities approach also attempts to minimize total costs but when assuming that reliability data is not fully known, which drives a slightly higher network investment cost (in comparison with the fixed probabilities approach) in order to efficiently hedge the solution and thus decrease risks, as demonstrated in Table I through the $\text{CVaR}_{99\%}$ and LOLE ($\text{CVaR}_{99\%}$ corresponds to SPS curtailment cost on average in the worst 1% events and LOLE is calculated across the year following the general method described in [43]).

TABLE II
SENSITIVITY ANALYSIS

Item	0% interval	±10% interval	±30% interval	±100% interval
Investment cost [million $]	17.7	17.7	19.1	19.1
Operating cost (planned, pre-fault) [million $]	283.7	283.8	283.5	283.4
Reserve holding cost (all services) [million $]	12.5	12.8	12.7	13.5
DER holding cost (all services) [million $]	15.3	15.4	15.3	15.5
Expected costs of generation reserves	0.22	0.22	0.22	0.22
DER	0.66	0.66	0.64	0.63
SPS [million $]*	4.00	3.65	2.75	2.16
Total cost [million $]	334.1	334.2	334.2	334.5

*Results obtained from an out-of-sample analysis of 30 million scenarios (3,000 random vectors where each vector element contains the failure rate of a system component uniformly distributed in the [−30%, 30%] ambiguity interval with respect to its reference value, times 10,000 contingencies (beyond $n-1$) for each of these vectors, considering exponentially distributed outages as indicated in [43]).

We have undertaken a sensitivity analysis in order to study the differences among various solutions determined by using different intervals applied on outage rates (e.g. 0%, ±10%, ±30%, ±50%, ±70%, ±90%, ±100%). These results are shown in Table II. As expected, the larger the interval, the more conservative the solution. In fact, the network investment solution associated with an interval of ±10% is equal to that obtained by running the model with fixed probabilities (i.e. 0% interval), which leads to the larger cost of network investment. Also, for intervals higher than ±30%, we observed more network investment (than that of the case with ±10% interval), but no differences in network investments within the entire range between ±30% and ±100%. Significant changes in the investment propositions can be observed, however, in both n-1 solutions (improved and traditional), which are equivalent to use the complete 0-1 ambiguity interval (with and without DER, respectively). This demonstrates that, from a network investment perspective, the results (in this case) obtained by the proposed probabilities interval approach are reasonably robust against changes in the length of the intervals used, but fundamentally different from those classical solutions obtained through the application of the $n-1$ security approach and the fixed probabilities approach.

V. 118-Busbar System Case Study

A. Input Data

This section demonstrates the scalability of our distributionally robust approach to DER. To do so, we apply it on the IEEE 118-busbar test system presented in [44]. We add 1300 MW of extra demand so as to increase congestion levels and consider 10 candidate lines. Regarding reliability data, reference outage rates are 0.001 occ/h and 0.00011 occ/h for generators and lines (every 100km), respectively considering a ±30% ambiguity interval. A total of 10 buses provide DER security services in a similar way as in the previous IEEE RTS case study.

TABLE III
RESULTS OF THE 118-BUSBAR SYSTEM CASE STUDY

Item	Case #1	Case #2	Case #3	Case #4
Number of time blocks	10	20	10	10
Number of snapshots	2	2	3	2
Serial or parallel	Parallel	Parallel	Parallel	Serial
Execution time [min]	177	289	353	861
Maximum RAM used [GB]	14	16	15	4
Investment cost [million $]	0.84	0.84	0.72	0.84
DER holding cost (all services) [million $]	10.19	10.13	10.38	10.19

B. Results and Discussion

Table III demonstrates the scalability of our modelling approach against different volumes of data. In effect, 10 and 20 demand levels (or blocks) are considered across a year. Also, we demonstrate that the time resolution in the post-fault conditions can be improved in order to more accurately represent the post-contingency evolution of demand services. These results also demonstrate the advantages of parallel computing, reducing computational time by almost 5 times, although RAM memory resources are increased by more than 3 times. In this particular case, Table III shows that investment decisions are more sensitive to post-fault rather than pre-fault time resolution, which demonstrates the importance of modelling appropriately the evolution of different DER services during post-contingency conditions in transmission network investment, as proposed in this paper.

VI. Conclusions

This paper proposed a distributionally robust approach to network security for planning future network infrastructure that can properly recognize both the participation of DER security services and limited knowledge of the underlying process behind the realization of system contingencies. To do so, we proposed a two-stage optimization model, where the first stage determines the transmission expansion plan and the scheduling of generation, up- and down-spinning reserves, and availability of DER post-contingency services, and the second stage minimizes the expected cost of corrective actions under various contingencies. Overall, the proposed model is capable to solve the TEP problem while simultaneously comprising several probability distributions of failure rates, necessary to properly determine the right portfolio of demand-based security services.

Through a number of quantitative assessments, we demonstrated the benefits of security services provided by DER and the advantages of our proposed interval probabilities approach against alternative $n-1$ security and fixed probabilities solutions. In particular, we demonstrated that while the $n-1$ approach significantly undermines the value of DER in displacing network capacity, the fixed probabilities counterpart is optimistic. In this vein, the interval probabilities approach properly utilizes DER services to displace network investments (and other security services from generation reserves), while providing hedged and secured solutions against the partially (un)known reliability data available in reality. Importantly, uncertainty in DER and how this may affect network investment, is proposed as further research.

REFERENCES

[1] P. Joskow, "Patterns of transmission investments," in *Competitive Electricity Markets and Sustainability*, F. Lévêque, Ed. Northampton, MA, USA: Edward Elgar, 2006.

[2] K. Moslehi and R. Kumar, "A reliability perspective of the smart grid," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 57–64, Jun. 2010.

[3] Y. Wang, I. R. Pordanjani, and W. Xu, "An event-driven demand response scheme for power system security enhancement," *IEEE Trans. Smart Grid*, vol. 2, no. 1, pp. 23–29, Mar. 2011.

[4] J. Aghaei and M. I. Alizadeh, "Robust n-k contingency constrained unit commitment with ancillary service demand response program," *IET Gener., Transmiss. Distrib.*, vol. 8, no. 12, pp. 1928–1936, 2014.

[5] P. Palensky and D. Dietrich, "Demand side management: Demand response, intelligent energy systems, and smart loads," *IEEE Trans. Ind. Informat.*, vol. 7, no. 3, pp. 381–388, Aug. 2011.

[6] A. Moreira, D. Pozo, A. Street, and E. Sauma, "Reliable renewable generation and transmission expansion planning: Co-optimizing system's resources for meeting renewable targets," *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 3246–3257, Jul. 2017.

[7] H. Zhang, V. Vittal, G. T. Heydt, and J. Quintero, "A mixed-integer linear programming approach for multi-stage security-constrained transmission expansion planning," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 1125–1133, May 2012.

[8] J. Choi, T. D. Mount, and R. J. Thomas, "Transmission expansion planning using contingency criteria," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2249–2261, Nov. 2007.

[9] A. K. Kazerooni and J. Mutale, "Transmission network planning under security and environmental constraints," *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 1169–1178, May 2010.

[10] M. Carrión, J. M. Arroyo, and N. Alguacil, "Vulnerability-constrained transmission expansion planning: A stochastic programming approach," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1436–1445, Nov. 2007.

[11] J. Choi, T. Tran, A. A. El-Keib, R. Thomas, H. Oh, and R. Billinton, "A method for transmission system expansion planning considering probabilistic reliability criteria," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1606–1615, Aug. 2005.

[12] W. Li, *Probabilistic Transmission Systems Planning*. New York, NY, USA: Wiley, 2011.

[13] R. Moreno, D. Pudjianto, and G. Strbac, "Transmission network investment with probabilistic security and corrective control," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3935–3944, Nov. 2013.

[14] G. Strbac, D. Kirschen, and R. Moreno, "Reliability standards for the operation and planning of future electricity networks," *Found. Trends Electr. Energy Syst.*, vol. 1, no. 3, pp. 143–219, 2016.

[15] A. L. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming," *Oper. Res.*, vol. 21, no. 5, pp. 1154–1157, Sep./Oct. 1973.

[16] A. Ben-Tal and A. Nemirovski, "Robust solutions of uncertain linear programs," *Oper. Res. Letters*, vol. 25, no. 1, pp. 1–13, 1999. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0167637799000164

[17] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35–53, Jan./Feb. 2004.

[18] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," *Math. Program.*, vol. 99, no. 2, pp. 351–376, Mar. 2004.

[19] D. Bertsimas, D. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Rev.*, vol. 53, no. 3, pp. 464–501, 2011. [Online]. Available: https://doi.org/10.1137/080734510

[20] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, Feb. 2013.

[21] B. Hu and L. Wu, "Robust SCUC considering continuous/discrete uncertainties and quick-start units: A two-stage robust optimization with mixed-integer recourse," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1407–1419, Mar. 2016.

[22] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 800–810, May 2012.

[23] Q. Wang, J.-P. Watson, and Y. Guan, "Two-stage robust optimization for N-k contingency-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2366–2375, Aug. 2013.

[24] S. Dehghan, N. Amjady, and A. J. Conejo, "Adaptive robust transmission expansion planning using linear decision rules," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 4024–4034, Sep. 2017.

[25] J. Li *et al.*, "Robust coordinated transmission and generation expansion planning considering ramping requirements and construction periods," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 268–280, Jan. 2018.

[26] R. A. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4558–4567, Nov. 2013.

[27] E. Delage and Y. Ye, "Distributionally robust optimization under moment uncertainty with application to data-driven problems," *Oper. Res.*, vol. 58, no. 3, pp. 595–612, 2010. [Online]. Available: https://doi.org/10.1287/opre.1090.0741

[28] J. Goh and M. Sim, "Distributionally robust optimization and its tractable approximations," *Oper. Res.*, vol. 58, no. 4-part-1, pp. 902–917, 2010. [Online]. Available: https://doi.org/10.1287/opre.1090.0795

[29] W. Wiesemann, D. Kuhn, and M. Sim, "Distributionally robust convex optimization," *Oper. Res.*, vol. 62, no. 6, pp. 1358–1376, 2014. [Online]. Available: https://doi.org/10.1287/opre.2014.1314

[30] C. Zhao and R. Jiang, "Distributionally robust contingency-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 94–102, Jan. 2018.

[31] Q. Bian, H. Xin, Z. Wang, D. Gan, and K. Wong, "Distributionally robust solution to the reserve scheduling problem with partial information of wind power," *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2822–2823, Sep. 2015.

[32] P. Xiong, P. Jirutitijaroen, and C. Singh, "A distributionally robust optimization model for unit commitment considering uncertain wind power generation," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 39–49, Jan. 2017.

[33] W. Wei, F. Liu, and S. Mei, "Distributionally robust co-optimization of energy and reserve dispatch," *IEEE Trans. Sustain. Energy*, vol. 7, no. 1, pp. 289–300, Jan. 2016.

[34] Y. Zhang, S. Shen, and J. L. Mathieu, "Distributionally robust chance-constrained optimal power flow with uncertain renewables and uncertain reserves provided by loads," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1378–1388, Mar. 2017.

[35] A. Bagheri, J. Wang, and C. Zhao, "Data-driven stochastic transmission expansion planning," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3461–3470, Sep. 2017.

[36] F. Alismail, P. Xiong, and C. Singh, "Optimal wind farm allocation in multi-area power systems using distributionally robust optimization approach," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 536–544, Jan. 2018.

[37] J. A. Schachter and P. Mancarella, "Demand response contracts as real options: A probabilistic evaluation framework under short-term and long-term uncertainties," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 868–878, Mar. 2016.

[38] N. Alguacil, A. Delgadillo, and J. M. Arroyo, "A trilevel programming approach for electric grid defense planning," *Comput. Oper. Res.*, vol. 41, no. 1, pp. 282–290, Jan. 2014.

[39] D. Alvarado, A. Moreira, R. Moreno, and G. Strbac, "Transmission network investment with distributed energy resources and distributionally robust security—auxiliary document," 2018. [Online]. Available: https://goo.gl/Uivxfe

[40] C. Grigg *et al.*, "The ieee reliability test system-1996. a report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999.

[41] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, “Matpower: Steady-state operations, planning, and analysis tools for power systems research and education,” *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12–19, Feb. 2011.
[42] “The Julia Language,” 2017. [Online]. Available: https://julialang.org/
[43] R. Billinton and R. Allan, *Reliability Evaluation of Power Systems*. New York, NY, USA: Springer US, 1996.
[44] University of Washington, “Power systems test case archive,” May 1993. [Online]. Available: https://www2.ee.washington.edu/research/pstca/, Accessed on: 2018-02-26.

Diego Alvarado received the B.Sc. degree in electrical engineering from the University of Chile, Santiago, Chile, in 2016. He is currently working toward the M.Sc. degree in electrical engineering with the University of Chile. His research interests include power systems economics, operation, and planning.

Alexandre Moreira (S’12) received the Electrical Engineering and Industrial Engineering degrees from the Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rio de Janeiro, Brazil, in 2011. He received the M.Sc. degree from the Electrical Engineering Department, PUC-Rio, in 2014. He is currently working toward the Ph.D. degree with the Department of Electrical and Electronic Engineering, Imperial College, London, U.K. His research interests include decision making under uncertainty as well as power system economics, operation, and planning.

Rodrigo Moreno (M’05) received the B.Sc. and M.Sc. degrees from Pontificia Universidad Catolica de Chile, Santiago, Chile, and the Ph.D. degree from Imperial College, London, U.K. He is currently an Assistant Professor with the University of Chile, Santiago, Chile, and a Research Associate with Imperial College London. His research interests include power system optimization, reliability and economics, renewable energy, and the smart grid.

Goran Strbac (M’95) is a Professor of electrical energy systems with Imperial College, London, U.K. His current research interests include electricity generation, transmission and distribution operation, planning and pricing, and integration of renewable, and distributed generation in electricity systems.