A Holistic Methodology to Identify Cost-Effective Smooth Routes for Power Transmission Lines

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***Abstract*—The design of cost-effective power line routes is a critical step towards elaborating a consistent network expansion plan. Although several attributes are considered by planning agents, the impact of sharp curvatures is typically neglected within the route design process, thus not accounting for the need for more expensive structures to support the power line. In this work, we tackle this issue by developing a holistic methodology that combines geoprocessing technologies with a novel combinatorial optimization technique to assist practitioners in identifying cost-effective smooth power line routes. For the latter, since the fundamental mathematical structure of the proposed tool falls into the class of a Quadratic Shortest Path Problem, an efficient dynamic programming algorithm is designed to handle the typical large-scale instances of power line tracing applications. The effectiveness of the proposed methodology is illustrated with a case study based on a real-world project of the Brazilian transmission sector.**

***Index Terms*—Dynamic programming, geographic information systems, geoprocessing techniques, power transmission line, smooth routing, transmission line expansion planning.**

I. Introduction

A RECURRENT challenge in most power systems around the globe is to plan and redesign its transmission network infrastructure seeking for an appropriate accommodation of the constantly growing demand level and variability in energy production within the daily system operation [1]. In the past years, this challenge, also known as Transmission Expansion Planning (TEP) problem, has been widely studied in technical literature with main focus on determining the set of transmission corridors that improves the system operation in the long run, with a least-cost expansion as primary objective [2], [3], [4]. In this context, due to its capital-intensive nature, it is critically important to evaluate a proper description of the power transmission line route within the candidate corridor and the respective implementation and construction costs in order to achieve a consistent expansion plan, both in technical and economical terms [5].

An accurate route evaluation of new power lines is a very complex task, which has been fundamentally tackled by mixing expert knowledge with limited spatial dataset [6], [7]. However, we argue that this current manual-based approach may not adequately assess the implementation and construction costs of the endeavour due to its intrinsic large scale characteristics. In fact, empirical evidence demonstrates that the initial estimation of line routes and costs differ significantly from the final ones. More precisely, from a technical perspective, the impact of multiple topographical aspects (e.g., nearby hydrology and hilly terrain) in the process of identifying the optimal transmission route and implementation costs need to be carefully evaluated. For instance, the varied terrain composition and potential richness of water bodies nearby the route significantly increase the degree of construction complexity. Additionally, social and environmental factors induce legal obstacles to the route design. Areas with environmental preservation, urban and traditional communities, for example, are government-protected. Therefore, it is often necessary to identify alternative detours on the tracing to avoid overrunning the administrative boundaries of these areas, which consequently affects the project's budget.

Due to this critical prospect in TEP initiatives, recent efforts have been made in technical literature to cost-effectively tackle and characterize the power transmission line routing problem. Among them, for instance, [6] adapts the well-known method called Structured Public Involvement (SPI), which, ultimately, combines community values, expert knowledge, and best engineering design techniques to route "acceptable-to-the-public" electric transmission lines. Furthermore, in [8], a spatial power network planning model is proposed, which co-optimizes, for a given set of candidate lines and cell linkages, the power network operation and electric line routing with main objective of minimizing the total transmission investment costs. Aiming at exploring the synergies and values of siting multiple power transmission lines within a single corridor, the authors in [9] analyzed and evaluated several metrics of overall impact of the preferred alternatives and spatial dissimilarities/variability of four methods in this framework. Results from a case study based

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on a project in Northeastern Alberta indicated that a range of Stakeholder values might distort the project weights (e.g., social-environmental and topographical factors), and, consequently, the maximum social-welfare outcome. More recently, [10] designed a two-step, multi-criteria procedure to analyze new electric transmission line projects. In a few words, the first step aims at forming the most reasonable estimates of new transmission lines, while the second step seeks to define, from the set of many analyzed lines at the first stage, a portfolio of the most appropriate and favorable areas for installing transmission lines. Finally, by adapting a heuristic known in technical literature as Rapid Random Tree, [11] presents a methodology for obtaining a transmission line route that considers diverse economic, social and environmental factors.

Notwithstanding the relevance of recent technical literature, the overall impact of sharp curvatures throughout the final route tracing, a critical aspect in new power line implementation and construction costs, has not been appropriately explored. More precisely, spatial route layouts that avoid intense curvatures (classified hereinafter as *smooth*) are preferable, since transmission towers that support these sharp trajectories require higher mechanical support, thus making the structure significantly more costly. As a result, the line trace commonly identified by planners and their respective construction costs are frequently far apart from the ones that will be actually implemented, which may delay or, in extreme cases, suspend the project. We also highlight that this planning inconsistency (i.e., the inconsistent cost perceived by the planner when solving the TEP problem, which may significantly differ from the real one) is worsened in the current context of most power systems around the globe since renewable power plants have been mostly constructed in locations far from the bulk system, thus necessitating new and cost-effective transmission infrastructure [12]. For instance, the United States' large on-shore renewable potential is in its mid-country, thus highly distant from major load centers [13]. Therefore, for an efficient carbon-averse generation expansion aiming at meeting the desired ambitious goals of decarbonization intended by regulators (e.g., a climate-neutral economy by 2050), multiple new long-distance high-voltage overhead transmission lines need to be cost-effectively built and properly estimated/anticipated in the planning process.

To tackle this issue, in this work, we combine state-of-the-art geoprocessing technologies from well known Geographic Information Systems (GIS) with mathematical programming and combinatorial optimization techniques in a holistic methodology in order to identify cost-effective, smooth routes for new power lines to support planners on the redesign and expansion of current transmission networks. From a methodological point-of-view, a three-dimensional map representing the region that covers the potential transmission line route is projected into a raster image (set of pixels associated with numerical values and spatial coordinates), considering terrain geographical characteristics. A graph is thus constructed by assigning each pixel in the image to a node in the graph and connecting them to their neighbors with an undirected arc. A source and target node are posteriorly assigned in the graph representing the physical location of the substations to be linked by the power transmission line; also a methodology to assign weight to each arc is developed, indicating local construction costs. These costs vary depending on topographical, social and environmental aspects. In order to efficiently consider the economic impact of intense curvatures in the route layout, the cost allocation of specific transmission tower structures for different levels of route curvatures is added to the cost composition by incorporating an adjacent arc selection penalty. We highlight, however, that this adjacent-penalty modeling induces a large-scale combinatorial optimization problem structure for which there is no computationally efficient methodology available to solve. Thus, in this work, an efficient algorithm for solving the resulting combinatorial problem based on the well-known dynamic programming algorithms known as Dijkstra and A^* [14], [15], [16] is adapted into the framework proposed in this work.

The effectiveness of the proposed methodology is illustrated with a case study based on a real-world project of the Brazilian transmission sector. A comparison between the standard approach for power line tracing and the proposed methodology is presented, contrasting, in detail, their spatial display and total construction costs. The main contributions of this work are threefold:

1) Propose a workable framework to incorporate deflection angles in the route design of a transmission line. To efficiently accommodate the economic and financial impact of intense curvatures in the route layout design, the associated cost of specific tower structures for different levels of deflection angles is added to the cost composition by means of an adjacent arc selection penalty. A Mixed-Integer Quadratic Programming (MIQP) problem with path-connectivity constraints is formulated to characterize the associated smooth power line routing identification problem.
2) Provide the mathematical tools to efficiently apply the framework in real-world transmission projects. More precisely, the structure of the smooth power line routing identification problem follows a MIQP problem, difficult to be solved for large-scale (real-world) instances using state-of-the-art algorithms or available solvers. Thus, an efficient least-quadratic-cost path-search algorithm with optimality guarantees is built upon the well-known shortest-path procedures known as Dijkstra and A^*.
3) Demonstrate the benefits of incorporating deflection angles in optimal transmission line routing through a case study of the Brazilian transmission sector. For this purpose, a real project that links the substations of *Poções III* and *Medeiros Neto II* through a single-circuit line operating at a 500 kV is considered. The standard approach for power line tracing and the proposed methodology are contrasted in terms of spatial display and total construction costs.

II. Optimal Smooth Power Line Routing Problem

Overhead Power Line Routing (PLR) problems are traditionally formulated as an instance of the weighted Shortest Path Problem (SPP) which aims, fundamentally, at identifying an optimal (cost-effective) path that connects two different nodes (e.g., substations) in a graph. The standard procedure (employed by practitioners and reported by technical literature) usually focus on co-minimizing the transmission line total length and construction costs, the latter characterized by social-environmental and topographical factors. However, the paths resulted from this standard procedure are oftentimes non-realistic from a technical perspective and non-directly implementable due to the presence of a large collection of sharp curves throughout the optimal path, necessitating thus of several special transmission towers to support these curvatures, making the total project significantly more costly than planned. To tackle this economic inconsistent planning issue, in this work, we propose an holistic methodology capable of endogenously taking into account the impact of sharp angles in the route design, hence identifying cost-effective smooth power line routes. For this purpose, we extend the fundamental structure of SPPs to take into account the impact of different adjacent arc selections, accounting for the power line trace angles. Methodologically, the proposed framework for designing smooth power line routes is formulated as a particular instance of the Quadratic Shortest Path Problem (QSPP) [17], [18] known as the Adjacent-QSPP (AQSPP) [19], which only considers interaction between arcs that have a common begin-node and ending-node. The use of such quadratic formulation allows a better representation of construction costs associated with route curvatures as nonlinear functions of the deflection angles. In the following subsection, we carefully describe the AQSPP structure for the optimal smooth power line route design proposed in this work.

A. Adjacent Quadratic Shortest Path Formulation

Let a weighted graph be defined by $G = (V, A)$, with V indicating the set of nodes in G and A the associated set of arcs. We assume a node $s \in V$ as the source node and $t \in V$ as the target node, such that a $s - t$ path $P = \{i, j, \ldots, k\}$ is an ordered set of nodes from $i = s$ to $k = t$. We define a linear cost function $c : A \to R_+$, which maps arcs into *linear* costs in the graph and a quadratic cost function $q : A \times A \to R_+$, which maps pair of arcs into *quadratic* cost in the graph. In particular, for the AQSPP, the quadratic costs of all non-adjacent pair of arcs are set to zero by construction, i.e.,

$$q_{(i,j),(k,l)} = 0, \quad \forall \big((i,j),(k,l)\big) \in A \times A \,|\, j \neq k. \tag{1}$$

Furthermore, to ease presentation, we define the predecessors and successors of each node $i \in V$ as the following node-to-set maps $\delta^-(i) = \{j \in V \,|\, (j,i) \in A\}$ and $\delta^+(i) = \{j \in V \,|\, (i,j) \in A\}$, respectively, and the arc adjacency list as $\delta(i) = \delta^-(i) \cup \delta^+(i)$. Finally, a binary variable $x_{(i,j)} \in \{0,1\}$ is defined to indicate the presence of arc $(i,j) \in A$ in the optimal path. In this context, when a given arc is selected to compose the optimal path, its associate linear cost component is incorporated into the power line total cost. Similarly, the selection of a pairwise linked adjacent arcs adds a quadratic cost to the power line total cost. By acknowledging (1), the Adjacent-QSPP formulation is presented in (2)–(4).

$$\min_{\mathbf{x}} \sum_{((i,j),(k,l)) \in A^2} x_{(i,j)} q_{(i,j),(k,l)} x_{(k,l)} + \sum_{(i,j) \in A} c_{(i,j)} x_{(i,j)} \tag{2}$$

subject to:

$$\sum_{j \in \delta^+(i)} x_{(i,j)} - \sum_{j \in \delta^-(i)} x_{(j,i)} = b_i, \qquad \forall\, i \in V; \tag{3}$$

$$x_{(i,j)} \in \{0,1\}, \qquad \forall\, (i,j) \in A, \tag{4}$$

with $\mathbf{x} \triangleq \{x_{(i,j)}\}_{(i,j) \in A}$ and $b_i \in \{-1, 0, 1\}$, such that $b_s = 1$, $b_t = -1$ and $b_i = 0$, $\forall\, i \in A \setminus \{s, t\}$. Structurally, (2)–(4) is a Mixed-Integer Quadratic Programming problem that aims at identifying a path that connects nodes $s \in V$ and $t \in V$ with the lowest combination of linear and quadratic costs, following the objective function (2). Equation (3) ensures path connectivity and feasibility through balance equations [16]. In the context of this work, the quadratic function $q_{(i,j),(k,l)}$ measures the economic impact of including in the optimal power line route the pair of sequential arcs $\big((i,j),(k,l)\big) \in A \times A \,|\, j = k$, thus accounting for the angles and its respective structural costs in the power line trace. As a consequence, by solving (2)–(4), the transmission expansion planner can co-optimize both the social-environmental and topographical factors (embedded into the linear costs) as well as the trace curvature, identifying thus a smoother power line route.

Following its mathematical structure, problem (2)–(4) is recognized as hard to be solved even for medium-sized graphs. In practice, however, real PLR instances are of non-negligible large-scale sizes, challenging thus the direct application of MIQP solvers or state-of-the-art algorithms to optimize (2)–(4). To overcome this computational issue for practical applications, in this work, a tailored path-search algorithm is adapted from well-known shortest path identification procedures known as Dijkstra and A^* algorithms. The algorithm is thoroughly presented in the next subsection.

B. Efficient Algorithm to Solve the AQSPP

Solving problem (2)–(4) is a challenging task for typical large-scale instances of real power line projects. Therefore, in this section, inspired by the ideas presented in [20], an efficient tailored algorithm to tackle this computational issue is adapted based on well-known shortest path procedures. More precisely, the proposed methodology is an extension of the well-known Dijkstra's and A^* algorithms [14], [15], [16] to accommodate the (adjacent) quadratic cost terms throughout the search process. For the sake of simplicity, hereinafter, we refer to the proposed solution procedure as Adjacent Quadratic A^* (or simply aqA^*). Roughly speaking, the algorithmic structure of the proposed aqA^* consists of two main steps, following a bidirectional search, namely:

1) an initial *backward* search considering only the linear counterpart of the MIQP problem (2)–(4) by running the standard Dijkstra algorithm, but starting from the target $t \in V$ to all other nodes in the graph, to find cost-to-go estimates;
2) a *forward* search with a variant of the standard A^* algorithm, from source $s \in V$ onward to the target $t \in V$ node, considering the cost-to-go estimates previously identified as lower bounds for the search.

Structurally, the bidirectional search procedure proposed in this work starts with a *backward* search using a standard Dijkstra algorithm starting on the target node $t \in V$, but only considering the linear counterpart of the MIQP problem (2)–(4), i.e., by setting $q_{(i,j),(k,l)} = 0, \forall \big((i,j),(k,l)\big) \in A \times A$. After completing the search and visiting all nodes in the graph, the smallest accumulated cost $(\{B_i\}_{i \in V})$ from $t \in V$ to every other node $i \in V \setminus \{t\}$ on the graph is stored by construction of the standard Dijkstra algorithm. These costs define costs-to-go estimates on the following steps of the aqA^* algorithm and are lower bounds for the original MIQP problem (2)–(4) since the non-negative-valued (*quadratic*) costs $q : A \times A \to \mathbb{R}_+$ are neglected in this *backward* search step. It should be highlighted that this lower-bound characteristic of the cost-to-go estimate guarantees that the aqA^* terminates with an optimal path for the AQSPP [21].

After performing this initial search and cost-to-go estimation, the second step of the solution procedure is launched by performing a *forward* search with a variant of the standard A^* algorithm from source node $s \in V$ to the target node $t \in V$ taking into account the cost-to-go estimates as lower bounds. More precisely, the variant has a similar structure to the standard A^*, but modifies how information and accumulated costs are defined. Formally, at each iteration of the search procedure, a new arc $(i,j) \in A$ is selected based on the current accumulated total cost to reach node $j \in V$ through $i \in V$, namely $d_{(i,j)}$, plus the addition of the costs-to-go estimate B_j (identified in the *backward* search step using only linear costs) to conclude the path towards $t \in V$. The latter term helps to enhance the computational performance of the algorithm by narrowing the size of the search tree [15]. These values are then used to select the next arc to process following a Dijkstra-related method. The aqA^* algorithm proposed in this work is summarized in Algorithm 1. We refer to [20] for a complete formal mathematical description.

Algorithm 1: Adjacent Quadratic A^* (aqA^*) Algorithm.

```
function aqA*(G, c, q, s, t):

// Initialization //
Set d_(s,s) ← 0 and d_(i,j) ← +∞, ∀ (i,j) ∈ A;
Set Q ← {(s,s)};

// Backward Search //
Set {B_i}_{i∈V} ← Dijkstra(G, c, t);

// Forward Search //
while Q ≠ ∅ do
    Solve (i,j) ∈ arg min_{(i,j)∈Q} {d_(i,j) + B_j};
    Set Q ← Q \ {(i,j)};
    for k ∈ δ(j) do
        if d_(j,k) > d_(i,j) + c_(j,k) + q_(i,j),(j,k) then
            Set d_(j,k) ← d_(i,j) + c_(j,k) + q_(i,j),(j,k);
            Set Q ← Q ∪ {(j,k)};
return d
```

III. General Framework for Power Line Routing With Angle Penalization

The process of construction of a new power transmission line and the respective connection into the grid generally begins at a planning stage few years prior to its actual deployment [5], [22]. At this stage, a sequence of technical-economical-environmental studies are performed, coordinated by a central planner (e.g., the responsible for the operation/regulation of the transmission system), specifying all the desired line requirements, with ultimate goal of identifying relevant connection candidates/alternatives for construction aiming at enhancing the system operation capability. Among this collection of studies, a critical one is precisely the specification of a guideline for the potential line routing and its expected total life-time expenditure (e.g., construction, operation and maintenance costs). In Brazil, for instance, as part of the auction design and procedure for the concession of new transmission-related infrastructure, the Energy Research Office (EPE) provides the market a full report with the characteristics of the auctioned assets including an indicative routing along with its respective expenses [23], [24]. In this context, on the other hand, the line routing problem becomes also of critical importance to private/public companies to identify value-maximizing offers into the transmission-related auction [11]. More specifically, due to the one-shot, capital-intensive investment nature, these companies typically devise its own line routing proposal seeking to better estimate the construction/technical costs and measured social-environmental impacts, thus efficiently adjusting the final offer.

Traditional frameworks for Power Line Routing typically focus on social-environmental and topographical factors in its design. In this section, we extend this framework to also account for the economic impact of sharp angles. In summary, the proposed methodology follows three major steps: (i) building a construction cost map based on topographical and social-environmental constraints; (ii) setting up a penalty function for the deflection angles; and (iii) solving the routing problem (2)–(4) formulated as an Adjacent Quadratic Shortest Path Problem (AQSPP). Fig. 1 presents the general framework proposed in this work. In the next sections, a thorough description of the first two steps is presented, both critical for an adequate specification of the problem intake to be solved by the methodology discussed in Section II.

A. Construction Cost Map - Social-Environmental and Topographical Factors

The first step of the proposed holistic methodology is to build a *construction cost map*, i.e., a digital image in which each pixel is associated with a spatial region and a construction cost is estimated for this region. In the framework proposed in this work, this map will be used as the foundation to devise the optimal power line route and evaluate the asset's total construction cost. Typically, the spatial data used to build this type of map have two different representations, namely, matrix and vector. The first representation (matrix form) organizes the data as georeferenced matrices such that each area is divided as a rectangular grid of regular cells (pixels). Each cell is associated

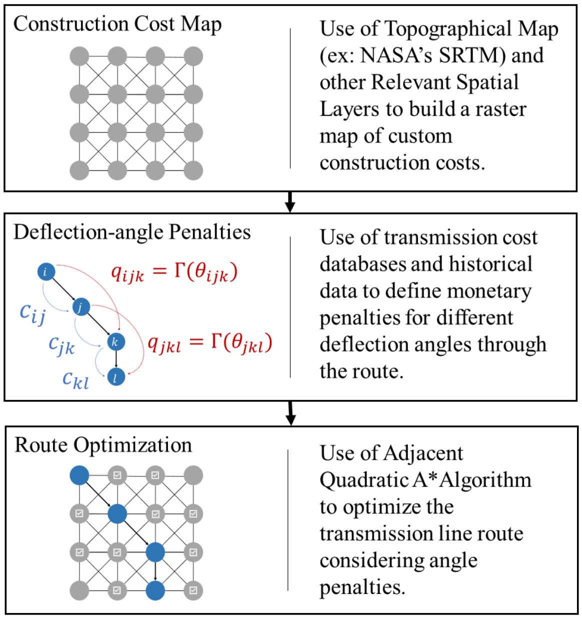

Fig. 1. General framework for power line routing with deflection-angle penalization.

with a unique coordinate and a numerical value. On the other hand, the latter model (vector form) represents elements as points, lines, or polygons. They are commonly used to model spatial regions that require greater precision, such as the route of a road or the administrative limits of a city. In this work, essentially, the considered construction cost map is composed of a topographical map (matrix form), required to provide data towards terrain relief, with a set of spatial constraints (vectors).

From the environmental perspective, Conservation Units can be classified as spatial territories protected by the government, due to their limited or special natural resources. To overlook the area and guarantee its protection, every unit usually has a government agency responsible for its administration and supervision. Depending on the trace of the power line nearby the area, costs can vary and increase substantially, forcing the route to contour it. For example, costs related to environmental authorizations or construction particularities (e.g., higher towers) can be so significative, that avoiding the protected area might be cheaper than trespassing it. Moreover, defining routes through water bodies requires special infrastructure and substantially increases the construction budget. Crossing wide water bodies, for instance, requires much higher towers than normal, which are very expensive and harder to set up. A distinguished case in this context, for instance, is the Brazilian Transmission Line called *Tucuruí-Macapá-Manaus*, which crosses the Amazon River margin to margin, where conductor cables are supported by two towers, 2.5 kilometers far from each other, and 300 meters high (similar in height to the Eiffel Tower).

On the social aspect, some communities (typically described as *traditional communities*) have their territory legally delimited and protected by the government and regulatory legislation. Interfering in these communities, such as indigenous territories, are of crucial concern in the licensing process by environmental agencies. Rural settlements, on the other hand, also have a great impact on routing. It generally refers to small rural properties and high habitational density in rural areas. Building power lines in these areas have been recognized as extremely challenging for transmission companies. Not only physical risks for residents should be taken into consideration, but the line right-of-way can interfere with production activities and may affect the community income flow. In the process of defining the overhead power line route, urban areas must be strongly avoided. Building transmission lines close to habitational concentration areas is challenging, expensive, and offers risk to residents. Urban growth is also a factor to be considered since an unanticipated urbanized region through original tracing can implicate the need of redesigning the power line tracing, thus increasing the project expenses.

Once the social-environmental factors are selected, the topographical map is set as the base map and all other relevant spatial areas are layered on top, delimiting areas of interest. Firstly, the topographical map is converted into a distance map taking into account the different terrain levels and altitudes. More precisely, the terrain slope can be described as the inclination of a given area and can be used to approximate linear distances. Then, the distance map is converted into a base cost map, assuming a reference construction cost (e.g., in k$/km). Finally, additional costs are added to the base cost map depending on the costs of different spatial areas.

Formally, in broad terms, the (linear) cost distribution along the construction map can be divided into two general categories: arc-wise *terrain-slope* ($c^{(\mathrm{TS})}_{(i,j)}$) and *non-geographic* ($c^{(\mathrm{NG})}_{(i,j)}$) costs, which maps two connected nodes $(i,j) \in A$ of the graph; and node-wise *social-environmental* ($c^{(\mathrm{SE})}_{i}$) cost, that reflects the local cost impact in a node $i \in V$ of the graph. Given this parameterization, the linear cost distribution can be specified as follows:

$$c_{(i,j)} = \varphi_{(i,j)} \left[c^{(\mathrm{TS})}_{(i,j)} + c^{(\mathrm{NG})}_{(i,j)} + \frac{c^{(\mathrm{SE})}_{i} + c^{(\mathrm{SE})}_{j}}{2} \right], \qquad (5)$$

with $\varphi_{(i,j)} = 1$, for transversely-connected nodes $(i,j) \in A$ and $\varphi_{(i,j)} = \sqrt{2}$ for diagonally-connected ones $(i,j) \in A$ [25]. Fig. 2 visually summarizes the full process of building the construction cost map.

B. Deflection Angle Penalties

Roughly speaking, power lines are simply electric towers connected by conductor cables. Therefore, depending on the spatial tower positioning within the trace, high inflection angles might occur. In this context, the greater the resulting angle, the stronger support structures must be used to handle the increased mechanical effort. These structures are often very expensive and can significantly impact construction costs. In this section, we carefully describe how these typically neglected construction costs are specified in the holistic methodology proposed in this work and its connection with the smooth power line routing model presented in Section II. Formally, for two adjacent arcs $\big((i,j),(j,k)\big) \in A \times A$, the respective adjacent (quadratic) cost $q_{(i,j),(j,k)}$ can be accounted by a deterministic function Γ of the

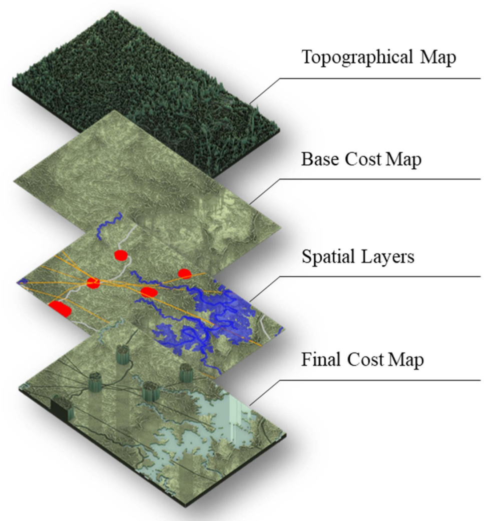

Fig. 2. Building the construction cost map by specifying social-environmental and topographical factors.

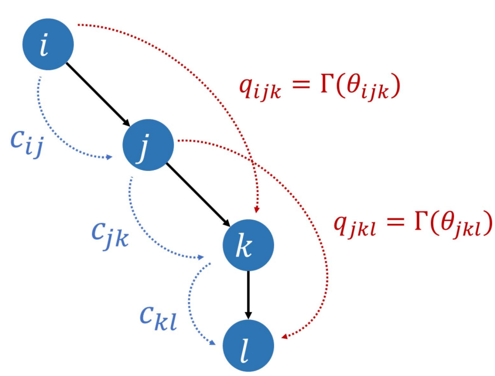

Fig. 3. Set of adjacent arcs and their associated linear and quadratic costs dynamics, the latter illustrated as a function of the deflection angles θ.

inflection angle $(\theta_{(i,j),(j,k)})$ between them, i.e.,

$$q_{(i,j),(j,k)} = \Gamma\Big(\theta_{(i,j),(j,k)}\Big). \tag{6}$$

Following this definition, $q_{(i,j),(j,k)}$ is null whenever there is no inflection between the adjacent arcs $\big((i,j),(j,k)\big) \in A \times A$, and $q_{(i,j),(j,k)}$ is strictly greater than zero, otherwise, and proportional to the angles formed by $\big((i,j),(j,k)\big) \in A \times A$. For expository purposes, Fig. 3 illustrates a set of adjacent arcs and their associated linear and quadratic costs dynamics.

A reasonable approach to specifying angle-related penalty functions would be to assign costs in terms of tower structure requirements. Each type of electric tower is designed to support a different number of cables, mechanical tensions, and inflection angle ranges [26]. For instance, Table I showcases the properties of the most common electric towers, mapping angle supports to total and relative costs. These cost ratios are relative to the costs of a Tangent Structure, the simplest and cheapest structure, used in segments with little to no deflection angles.

TABLE I
PROPERTIES OF THE MOST COMMON ELECTRIC TOWERS, MAPPING ANGLE SUPPORTS TO TOTAL AND RELATIVE COSTS

Structure	**Angle Range**	**Total Cost** [$k\$$]	**Relative Cost** [%]
Tangent	0º to 2º	176.6	1.00
Strain	2º to 45º	385.4	2.18
Dead-end	$>$ 45º	551.6	3.12

The construction cost map and angle-penalty functions are then used to build a weighted graph with linear and quadratic costs, suitable for the direct application of the methodology presented in Section II. More specifically, the resulting graph is an abstract data type directly related to the cost map, in which each pixel becomes a node and their neighbors are connected through weighted arcs. For every pair of connected arcs, the angle-penalty function is applied to define the quadratic cost according to the angles formed by the arcs. Furthermore, the social-environmental, non-geographic, and topographical factors are accounted by the linear costs as previously discussed in Section III-A (we refer to Appendix A for a detailed roadmap of the cost distribution construction). Once the cost structure is prepared, the aqA^* Algorithm discussed in Section II-B is applied to efficiently identify the smoother power line route. In the next section, a numerical experiment based on a real project of the Brazilian power transmission sector is presented to illustrate the applicability of the holistic methodology proposed in this work.

Remark I: It is worth highlighting that, by specifying angle-related penalty functions based on costs in terms of tower structure requirements, the associated quadratic term $\boldsymbol{q} \triangleq \{q_{(i,j),(k,l)}\}_{((i,j),(k,l))\in A^2}$ can be dynamically evaluated along with the execution of the aqA^* algorithm. In this context, large *a priori* computationally demanding calculations and non-negligible sets of memory allocation are avoided, enhancing, thus, the solution capability of the proposed methodology.

IV. NUMERICAL EXPERIMENT

To highlight the potential benefits in endogenously considering the impact of different levels of curvatures in the power line routing process, in this section, a numerical experiment is presented based on a real project under construction evaluation of the Brazilian power transmission sector. The chosen transmission line is a single-circuit between substations *Poções III* and *Medeiros Neto II* operating at a 500 kV Voltage level. The line crosses two major Brazilian states – Bahia and Minas Gerais, and is planned to improve the power transfer capability between the Northeast and Southeast Regions of the country, critical to flow the recently built large amount of renewable capacity to the main load center in the country [27]. More specifically, most of the power demand in Brazil is located in the Southeast region, at which the biggest cities and most advanced industrial parks are located. On the other hand, the Northeast region will accommodate most of the country's generation expansion for the next years (having as the main driver the renewable power) mainly due to the significant drop in solar and wind technology prices and favorable weather conditions, attracting worldwide

TABLE II
LIST OF CONSIDERED SPATIAL CONSTRAINTS AND ASSOCIATED COSTS ASSUMPTIONS

Spatial Constraint	Source	Year	Additional Cost [mi$/km]
Topographical Map	Embrapa	2005	1.23
Urban Areas	IBGE	2018	9.44
Water Bodies	IBGE	2015	12.55
Indigenous Land	INCRA	2015	∞
Conversation Units	INCRA	2017	∞
Rural Areas	INCRA	2017	4.92
Roads	Embrapa	2018	0.00
Railways	Embrapa	2018	0.00
Transmission Lines	EPE	2021	0.00
Military Areas	IBGE	2019	∞
Archaeological Sites	IPHAN	2020	∞
Possible Caves	CECAV	2019	∞
Aerodromes	PLNT	2019	∞

investors and big generation projects. According to the official expansion plan of Brazil, the *Poções III–Medeiros Neto II* power line is expected to start its operation in 2026 [24]. The substation *Poções III* is named after the city Poções located in the state of Bahia and *Medeiros Neto II* is also named after a city, Medeiros Neto, which borders the Southeast state of Minas Gerais. Studies from the Brazilian Energy Research Office (EPE) describes the candidate line with the following technical and economical properties:

- Conductor: 6 x TERN (795 MCM)
- Line Reactance: 0.0140 [Ω/km]
- Line Resistence: 0.1917 [Ω/km]
- Flow Capacity: 1129 [MVA]
- Emergency Flow Capacity: 1267 [MVA]
- Instalation Cost: 1.234 [mi R$/km]

These parameters and cost estimation comes from different planning studies performed by the official agency. They are divided into 5 reports, covering technical, economical, and environmental detailed analysis. Roughly speaking, the planning process starts with an initial evaluation of power line candidates and the definition of the respective power corridors, comprising a 10 to 20-kilometer area surrounding a preliminary route. Later on, technical analysis is performed to detail the social-environmental factors within the selected corridor and officially propose a potential route for the power line. The holistic methodology proposed in this work covers the afore-described planning process and improves on the representation of technical constraints, thus providing more realistic routes with a better assessment of its construction costs.

Following the detailed process described in Section III and Appendix A, the proposed methodology begins with the selection of the spatial data of the studied area at which the power line will pass through. Initially, a tool called *EPE Web Map*[1] was used to cross spatial layers of transmission infrastructure and social-environmental constraints. Further on, existing reports from the planner were also used to filter data. Table II compiles the list of public data collected and assumptions for additional costs. The values set to ∞ refer to spatial areas at which the route is forbidden to go through.

[1]EPE Web Map – https://www.epe.gov.br/en/publications/publications/webmap-epe.

The quadratic costs are defined according to Table I and assigned dynamically within the algorithm, following Remark I in Section III-B. For comparability purposes, in this section, we benchmark the proposed methodology with the solution of the linear counterpart of the problem (2)–(4) (i.e., by setting $q_{(i,j),(k,l)} = 0, \forall \big((i,j),(k,l)\big) \in A \times A$), following traditional standards employed by industry practices and academic works. We refer to the latter as *SPP Route*. Fig. 4 displays the optimal power line trace of both *AQSPP Route* (based on the proposed methodology) and *SPP Route*, along with the cost map and spatial constraints' layers. Structurally, the graph instance built for this problem has a spatial resolution of 30 × 30 meters, resulting in 15,497,220 nodes, 61,952,021 undirected arcs and 991,232,336 quadratic arcs. Both algorithms were executed in an Intel Core i7-10700 K 4.8 GHz with 64 GB of RAM. The total solution-time by running the aqA^* algorithm to this instance was roughly 4150 seconds, suitable for planning stages. Furthermore, it also worth mentioning that commercial MIQP solvers was not able to handle problem (2)–(4) for this instance size (we refer to Subsection IV-A for a thorough computational analysis and discussion).

First, we highlight the general difference in route trajectories, by comparing the *AQSPP Route* (blue line) with the benchmark *SPP Route* (red line). More precisely, we should notice the similarities in the departure and arrival points, and the differences in the middle section of the trajectory. In fact, the spatial distribution of the construction costs gives us intuition towards the routing decisions. The *AQSPP Route* chooses a detour seeking a cheaper region (bluish color), where it can build longer sections of straight lines, benefiting from lower construction costs and curvature penalties. Further on, we can also notice long diagonal movements throughout *AQSPP Route*, particularly in the more expensive region (reddish color) at the bottom section of the cost map. This choice of power line route underlines the ability of the proposed methodology to avail lower costs from a valley region while also minimizing curvature penalties.

Furthermore, we evaluate the routing solution in more aggregated maps in terms of curvature-aversion. For this purpose, we aggregate the base-case spatial resolution of 30 × 30 meters into resolutions of 150 × 150, 300 × 300 and 450 × 450 meters. Aggregation techniques are often used by practitioners for map reduction, aiming at speeding up the search in shortest path problems. Since they reduce the map size, the map conversion results in fewer nodes and arcs. Table III summarizes the results for each power line trace approach and spatial resolution. The curvatures of each power line were calculated based on direction shifts on the route, where each direction shift resulted in a specific deflection angle (0°, 45° or 90°), following the tower structure of Table I. The angle occurrences were counted along the routes and presented as percentiles. Roughly speaking, this metric quantifies the curvature-aversion of a given route. Note that incorporating penalties for deflection angles significantly reduced high-angle deflections for all map resolutions. This is particularly evident for the base-case of 30 × 30 meters resolution. The contrast with *SPP Route* shows almost six times

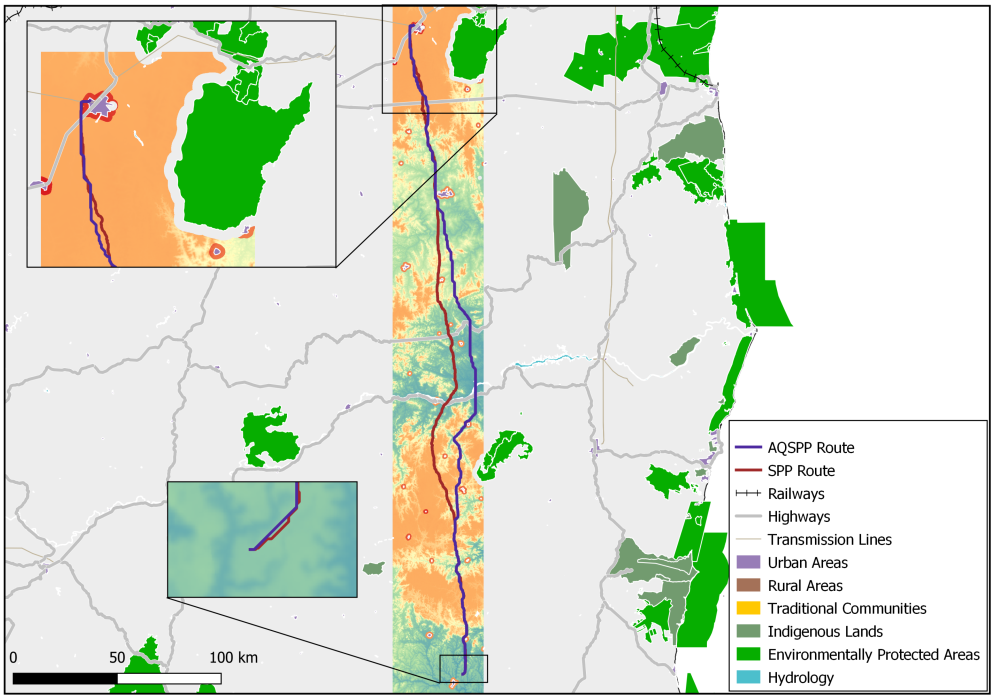

Fig. 4. Optimal power line trace of both *AQSPP Route* (blue line) and *SPP Route* (red line), along with the cost map and spatial constraints' layers.

TABLE III
ANGLE DEFLECTION OVERVIEW

Algorithm	**Grid [m]**	**0º**	**45º**	**90º**
SPP Route	*30 × 30*	84.0%	15.3%	0.7%
AQSPP Route	*30 × 30*	95.2%	4.5%	0.4%
SPP Route	*150 × 150*	70.7%	6.6%	22.7%
AQSPP Route	*150 × 150*	73.9%	17.4%	8.7%
SPP Route	*300 × 300*	40.8%	41.9%	17.3%
AQSPP Route	*300 × 300*	72.4%	21.0%	6.6%
SPP Route	*450 × 450*	32.8%	45.8%	21.4%
AQSPP Route	*450 × 450*	66.9%	26.6%	6.5%

TABLE IV
COMPUTATIONAL TIME (IN SECONDS) REQUIRED BY EACH PROCEDURE (aqA^* AND MIQP) TO SOLVE THE ADJACENT QUADRATIC ROUTING PROBLEM FOR DIFFERENT GRID RESOLUTIONS

Grid Resolution [m]	**Number of Arcs [Binary Variables]**	aqA^* **[sec.]**	**MIQP [sec.]**
1800 × 1800	16,763	0.09	5.80
900 × 900	66,831	0.57	432.32
450 × 450	275,558	4.18	2,405.33
300 × 300	617,507	13.63	10,352.49
120 × 120	3,867,806	187.89	-
30 × 30	61,952,021	4,150.00	-

fewer occurrences of 45° angles and a reduction of 90° angles by an order three. Also, over 95% of line segments are associated with straight lines (0° angle). The same pattern is exacerbated for lower map resolutions. By decreasing the map granularity, the *SPP Route* significantly increases the relative amount of large deflection angles; meanwhile, the proposed methodology is able to maintain the number of deflection angles in the power line trace at low granularity levels.

A. *Computational Performance Analysis*

In order to illustrate the solution capability of the proposed methodology, in particular the computational performance of the path-search procedure described in Section II-B, in this section, a computational analysis is performed, benchmarking the aqA^* algorithm against directly solving the MIQP problem (2)–(4) formulated in Section II-A. For consistency in presentation, the set-up for the analysis is based on the same case and instance discussed in the previous section, but varying the map resolution, thus changing the size of the optimization problem, in particular, the number of binary variables. Furthermore, we focus the analysis on the computational performance of both solution methods (MIQP and aqA^*) rather than optimality aspects since, by construction, both achieve an optimal path. The MIQP problem was implemented in Julia/JuMP and solved using Gurobi Solver 8.0 under the same machine set-up described in the previous section.

Table IV showcases the computational time, expressed in seconds, required by each procedure considered in this analysis (aqA^* in column 3 and MIQP in column 4) to solve the adjacent quadratic routing problem for different grid resolutions (column 1). For expository purposes, we also quantify in column 2 the number of resulting arcs for the given map resolution, which directly specifies the amount binary variables in the problem. Firstly, note that the computing time for the MIQP benchmark rapidly increases for low-scale map resolutions. For instance, a 450 × 450 resolution (row 4 in Table IV) requires roughly 40 minutes to be solved by the MIQP formulation, meanwhile the

aqA^* algorithm could handle the same instance in few seconds. Furthermore, for instances with higher resolution than 300 $\times$ 300, the MIQP benchmark could not manage the problem. On the other hand, the path-search procedure proposed in Section II-B was able to solve all considered resolutions, including the thinner one of 30 $\times$ 30 meters in a reasonable computational time.

V. Conclusion

We present a workable framework to incorporate deflection angles in route optimization for power lines. A combination of geoprocessing technologies with a novel mathematical programming and combinatorial optimization technique is designed to efficiently find the optimal route. For the latter, specifically, a Mixed-Integer Quadratic Programming problem with path-connectivity constraints is proposed to account for the economic and financial impact of high angles in the transmission line route. Moreover, in order to handle the MIQP for large-scale (real-world) instances, a quadratic least-cost path-search is designed, built upon the well-known dynamic programming algorithms known as Dijkstra and A^*. The proposed framework is applied to identify the cost-effective route of line 500 kV *Poções III—Medeiros Neto II*, part of the Brazilian transmission expansion plan. Results show a significant reduction in the amount of high-angle deflections for different test instances, thus potentially reducing the overall implementation and construction costs of the power line. The benefits are particularly evident on maps with greater spatial resolution.

Future works may extend the proposed methodology for related quadratic problems, such as the quadratic travelling salesman problem [28] and the quadratic vehicle routing problem [29]. Furthermore, evaluating the final routes of both the proposed methodology (*AQSPP Route*) and the standard industry practice benchmark (*SPP Route*) in a power transmission line design software might be of interest both to central planner and private agents, with ultimate goal of effectively estimating the total resulting expenses of the line. In addition, extending the framework by adding tower-sitting algorithms to improve the quality of the route should also be explored. More specifically, the extension of the here-proposed framework to identify cost-effective, smooth routes for new power transmission lines to also account for the tower location specification in a co-optimized set-up, thus designing a unified planning-deployment methodology, is of considerable importance both for central planner and private agents. Future research in this direction can be built upon the framework proposed in this work by adapting it, for instance, into the tower spotting methodology discussed in [22].

Appendix A
Roadmap for the Cost Distribution Construction

The process starts with a topographical map of the studied area, describing the terrain in terms of height. This map is a spatial matrix [30] in which each cell refers to a specific coordinate within the studied area and a height measurement of the surrounding space. Once the heights are known, the geographical distance between two neighbour coordinates are computed assuming linear terrain slopes using standard geometry laws [8]. The distance is then mapped into a reference construction cost (e.g., see the second line ("Topographical Map") of Table II for the reference construction cost used in the numerical experiment conducted in this work) and assigned as the cost category $c^{(\mathrm{TS})}_{(i,j)}$ for each arc $(i, j) \in A$.

Further on, social-environmental layers are included in the Construction Cost Map. More specifically, they are formatted as spatial vectors [30] and delimit geographical regions such as: urban areas, Indigenous land, conservation areas, and others (see rows 3–7 and 11–14 of Table II for a reference construction cost used in the numerical experiment conducted in this work). Each layer has its unique, node-specific, value $c^{(\mathrm{SE})}_i$ in terms of additional construction costs for building a line passing through the associated node $i \in V$. It is important to highlight that the costs are node-wise as a result of the overlaying process. Thus, the arc-wise costs are updated to $c^{(\mathrm{TS})}_{(i,j)} + \frac{c^{(\mathrm{SE})}_i + c^{(\mathrm{SE})}_j}{2}$, $\forall\,(i, j) \in A$.

Non-geographical costs $c^{(\mathrm{NG})}_{(i,j)}$ of each arc $(i, j) \in A$ should also be considered in the process. These costs are usually related to infrastructural layers as Federal roads, existing transmission lines and airports (see rows 8–10 of Table II for a reference construction cost used in the numerical experiment conducted in this work), for instance, and are added into the construction cost for each arc $(i, j) \in A$, resulting in $c^{(\mathrm{TS})}_{(i,j)} + c^{(\mathrm{NG})}_{(i,j)} + \frac{c^{(\mathrm{SE})}_i + c^{(\mathrm{SE})}_j}{2}$.

With these components, the final arc-wise linear costs $c_{(i,j)}$ must then be adjusted by a factor $\phi_{(i,j)}$ based on how the nodes are connected: transversely or diagonally [25]. Transversely-connected nodes are kept unaltered (i.e., multiplied by a factor $\phi_{(i,j)} = 1.0$), whereas Diagonally-connected nodes are multiplied by a factor $\phi_{(i,j)} = \sqrt{2}$. By following this process, a weighted-graph structure is built, setting the weights as estimates of the project construction (linear) costs. Finally, all pairs of adjacent arcs are mapped, along with their resulting deflection angles $\theta_{(i,j),(j,k)}$, and associated with a cost $q_{(i,j),(j,k)}$ based, for instance, in Table I as used in the numerical experiment conducted in this work.

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