Operations Research

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Unit Commitment Problem with Energy Storage Under Correlated Renewables Uncertainty

Felipe Cordera, Rodrigo Moreno, Fernando Ordoñez

To cite this article:
Felipe Cordera, Rodrigo Moreno, Fernando Ordoñez (2023) Unit Commitment Problem with Energy Storage Under Correlated Renewables Uncertainty. Operations Research 71(6):1960-1977. https://doi.org/10.1287/opre.2021.0211

OPERATIONS RESEARCH
Vol. 71, No. 6, November–December 2023, pp. 1960–1977
ISSN 0030-364X (print), ISSN 1526-5463 (online)
https://pubsonline.informs.org/journal/opre

Special Issue

Unit Commitment Problem with Energy Storage Under Correlated Renewables Uncertainty

Felipe Cordera,[a,b,*] **Rodrigo Moreno,**[a,c] **Fernando Ordoñez**[a,c]

[a] Faculty of Physical and Mathematical Sciences, University of Chile, 8330111 Santiago, Chile; [b] Operations Research Center, Massachusetts Institute of Technology, Cambridge, Massachusetts 02142; [c] Instituto Sistemas Complejos de Ingenieria, 8370398 Santiago, Chile

*Corresponding author

Contact: fcordera@mit.edu, https://orcid.org/0000-0001-6936-9656 (FC); rmorenovieyra@ing.uchile.cl, https://orcid.org/0000-0001-5538-445X (RM); fordon@dii.uchile.cl, https://orcid.org/0000-0001-7037-6300 (FO)

Received: March 31, 2021
Revised: July 13, 2021; February 11, 2023
Accepted: August 28, 2023
Published Online in Articles in Advance: October 27, 2023

Area of Review: Special Issue on Computational Advances in Short-Term Power System Operation

https://doi.org/10.1287/opre.2021.0211

Abstract. The extensive integration of renewable generation in electricity systems is significantly increasing the variability and correlation in power availability and the need for energy storage capacity. This increased uncertainty and storage capacity should be considered in operational decisions such as the short-term unit commitment (UC) problem. In this work, we formulate a day-ahead UC problem with energy storage, considering multistage correlated uncertainty on renewables' power availability. We solve this multistage stochastic unit commitment (MSUC) problem with integer variables in the first stage using a new variant of SDDP that can explicitly deal with temporal correlations. Our computational results on the IEEE 118-bus system demonstrate the significance of considering multistage uncertainty and correlations, comparing our solution with other multistage solutions, two-stage solutions, and deterministic solutions typically used by industry. We also solve the MSUC problem for a representation of the Chilean power system, finding superior UC solutions for scenarios where adapting generation to the unfolding uncertainty is costly. Finally, we demonstrate that the MSUC approach can be used to define a more efficient deterministic UC solution, outperforming the current industry practice.

History: This article is part of the *Operations Research* Special Issue on Computational Advances in Short-Term Power System Operation.

Funding: This work was supported by the Instituto de Sistemas Complejos de Ingeniería [Grants ANID AFB 220003, FONDECYT 1201844, and FONDECYT 1231924].

Keywords: energy • natural resources • stochastic programming • large-scale systems • linear programming

1. Introduction

Because of climate change, various governments around the globe are committed to decarbonizing the energy sector. This is being translated into policies that promote more extensive integrations of renewable generation in the electricity system. Various jurisdictions in the United States have committed renewables targets (the so-called renewables portfolio standards) by the next decade, hitting 50% (Maryland, New Jersey) and even 100% (Washington DC) of the participation of the retail sales from renewables (PJM 2020). Such renewables-centered policies are now being complemented with carbon neutrality objectives and plans to retire coal power units. Chilean authorities, for example, are planning to decommission all coal power plants in a decade and the country is already facing hours when solar and wind power generation supply more than half of the country's demand. Great Britain, in another example, ran coal-free for almost 60% of the time in 2020, with the renewables share already surpassing that of fossil-fuel generators. Remarkably, on a good day, wind can provide up to 60% of Great Britain's electricity demand (BBC News 2020, edie 2021).

Integrating more and more renewables will require installing supporting technologies that can provide the needed flexibility to deal with variable generation effectively. Of these technologies, energy storage is of utmost importance because it can store energy when renewable generation exceeds demand, delivering it later on when renewables do not suffice. However, managing a system featuring these characteristics is not trivial and is significantly more complex than operating a fully controllable, fossil-fueled generation fleet. Indeed, there is now a pressing need to appropriately recognize some of these characteristics (such as the stochastic nature of renewable energy and the presence of energy storage) in operational decision-making problems that must be solved by system operators to supply demand with appropriate quality and reliability levels. One of these operational problems is the so-called unit commitment (UC).

The UC problem refers to the optimal operational discrete decisions where the on/off statuses of generation units are determined to minimize the cost of meeting demand over a short (one day) planning horizon in a power system. The main versions of the UC problem are deterministic and stochastic, with industry traditionally using deterministic ones (Zheng et al. 2014); in Section 2, we provide a more comprehensive description and review of the UC problem. In its stochastic version, the uncertainty present due to demand and generation is commonly represented as a two-stage stochastic optimization problem, where the uncertainty for the next 24 hours is revealed at the beginning of the day (Ruiz et al. 2009, Papavasiliou and Oren 2013, Morales-España et al. 2018). However, the increasing integration of uncertain renewable resources and larger storage plants motivates the study of a multistage stochastic model for the UC problem.

In this work, we study the use of the stochastic dual dynamic programming (SDDP) approach to tackle a multistage stochastic UC (MSUC) problem with significant penetration of renewable generation (wind, solar) and energy storage (batteries, pumped-storage) capacities. The integration of renewable resources creates a generation mix that is highly variable and hard to predict accurately, with significant short-term correlation. Although SDDP is typically used to solve dispatch problems where binary/commitment decisions are already fixed, it does allow the consideration of first-stage binary variables. Therefore, it is a natural alternative for MSUC, helping to reduce the gap between real-time operations and day-ahead planning. Under this approach, units can be committed the day before, while considering dynamic operational decisions for the next 24 hours instead of trajectories of fixed decisions as in the two-stage formulation.

A limitation observed in the SDDP literature is the assumption of stagewise-independent uncertainty (Infanger and Morton 1996, De Queiroz and Morton 2013), which makes questionable its use with correlated renewable generation uncertainty. A recent extension, the Markov-SDDP method (Philpott and De Matos 2012), admits a broad range of dependency models at the expense of an increase in time and space complexity; nevertheless, it is not always entirely clear how to provide solution quality guarantees with respect to the "true" (also called unsampled or nondiscretized) stochastic problem (Löhndorf and Shapiro 2019). To ensure the latter, in this work, we rely on an importance sampling–based technique to generate the discretized scenarios required by Markov-SDDP (Cordera et al. 2021) and use this method to solve the MSUC problem.

We show that our approach is appropriate to model and solve MSUC in real-life settings. We evaluate the proposed model's benefits on a modified 118-bus IEEE test system and a 227-bus representation of the Chilean system, comparable to what the country's independent system operator (ISO) uses to decide daily UC (notice that the Chilean system features a predominantly linear/narrower network configuration; therefore, the methods discussed in this paper may require further adjustments for other network characteristics). The uncertainty corresponds to wind power generation profiles across different system locations, which exhibit significant space and, importantly for this paper, temporal correlations. In the IEEE test system, we compare the solutions obtained for the MSUC with temporal correlations (our approach) to four alternatives: MSUC without temporal correlations, two-stage stochastic UC with temporal correlations, two-stage stochastic UC without temporal correlations, and the deterministic approach with (exogenous) reserve requirements (corresponding to the industry standard). In the Chilean case study, we focus on how material the benefits of our approach are, in reality, by comparing it against the current deterministic UC used by the Chilean ISO. We also evaluate and discuss possible improvements to the current deterministic UC, by taking advantage of the information determined by our MSUC approach in terms of the reserves genuinely required by the system; thus, we propose to use the reserve volumes found by our MSUC as inputs in the deterministic UC model.

The main contributions of this work are as follows:

1. We use the SDDP method to solve a MSUC problem that makes day-ahead commitment decisions in line with current industry practice, while considering dynamic dispatch decisions. In this vein, this combination of model and solution method helps to improve consistency between real-time and day-ahead decisions in terms of considering multistage uncertainty on the next day, when determining the unit commitment solution day-ahead. Our computational results demonstrate, on a generic IEEE system and the Chilean system, the significant benefits of this approach compared with various alternatives, including that of standard use in the power industry.

2. We use an importance sampling–based technique to generate the scenario tree for the Markov-SDDP solution method, used to solve MSUC. This approach allows us to consider more realistic wind uncertainty models that have both spatial and temporal correlations and, in addition, complicated nonlinear time dependencies that are difficult to include in classic-SDDP solution methods.

3. Our results on the Chilean electricity system also show that the current deterministic UC approach (used by the ISO) is efficient at present, as it is not costly to adapt generation. However, going forward, it becomes inefficient with respect to the MSUC solution in scenarios with more wind uncertainty and dry (hydro) conditions.

4. We propose a deterministic UC approach, with reserve requirements determined from our MSUC approach. This new deterministic UC solution obtains similar efficiency levels to the MSUC approach in the

Chilean electricity system, even under high wind uncertainty and dry conditions, outperforming the current industry practice. We argue that introducing this new deterministic UC approach should be straightforward since no significant modification is needed in the ancillary services market.

This paper is organized as follows. Section 2 introduces the notation and problem statement, including a literature review. In Section 3, we present the proposed MSUC model. In Section 4, we describe the experimental setting in which we test our model. In Section 5, we show the computational results on both the IEEE test system and the Chilean system. Finally, we conclude in Section 6.

2. Notation and Problem Statement

We begin by presenting the UC problem for an electricity network with generation from thermal units and renewable energy utilities, particularly wind generation and pumped-storage units. We then review the SDDP algorithm used for optimizing operating decisions of this network when considering multistage uncertainty in wind power availability.

To fix notation, we present below the index sets, parameters and variables used Table 1–3.

2.1. UC Problem

UC is the problem determining the next-day schedule of generating units that should be synchronized/online (i.e., on/off status of power units) to meet load demand at minimum cost while satisfying physical constraints. To protect against outages of generating units and/or transmission lines and load forecast errors, reserve requirements are traditionally included in deterministic formulations. However, this may lead to conservative solutions and requires nontrivial tuning, as mentioned by Aminifar et al. (2009). Moreover, a significant presence of renewable energy makes it more difficult to estimate the necessary reserve requirements due to their high intrinsic uncertainty.

Table 2. Parameters

Notation	Description
Δt	Length of time periods
$d_d{}^t$	Demand of load d at period t
eb_l (sb_l)	End (start) bus of line l
B_l	Susceptance of line l
F_l	Flow capacity of line l
$\eta_p{}^{chg}(\eta_p{}^{disc})$	Charge (discharge) efficiency of pumped-storage unit p
$S_p{}^{min}(S_p{}^{max})$	Minimum (maximum) energy storage capacity of pumped-storage unit p
$Cap_p{}^{chg}(Cap_p^{disc})$	Charge (discharge) power capacity of pumped-storage unit p
Cap_w	Installed capacity of wind unit w
$I_w{}^t$	Observed wind power in wind unit w at period t
$P_g{}^{min}$ $(P_g{}^{max})$	Technical minimum (maximum) power capacity of thermal unit g
a_g, b_g, c_g	Parameters of quadratic cost function of thermal unit g
$c_g{}^{SU}(c_g{}^{SD})$	Start-up (shut-down) cost of thermal unit g
$R_g{}^{up}(R_g{}^{down})$	Ramp-up (ramp-down) rate limit of thermal unit g
$R_g{}^{SU}(R_g{}^{SD})$	Start-up (shut-down) ramp rate limit of thermal unit g
UT_g (DT_g)	Minimum uptime (downtime) of thermal unit g
$C_p{}^{gs}$ $(C_p{}^{ls})$	Penalty cost for exceeding generation (load shedding)

Approaches that explicitly model the uncertainty in the UC problem use either stochastic optimization or robust optimization. We refer the reader to Zheng et al. (2014) for an extensive review on this topic. For example, Papavasiliou and Oren (2013) presented a stochastic model to solve large-scale systems with component failures and high wind penetration, Jiang et al. (2011) proposed a robust approach to accommodate wind uncertainty in a system with pumped-storage units, and,

Table 1. Sets

Notation	Description
B	Buses
L	Transmission lines (including transformers)
D	Loads
G	Thermal units
W	Wind units
P	Pumped-storage units
T	Time periods
I	Indices of vertices in piecewise approximation
G_b	Thermal units at bus b
W_b	Wind units at bus b
P_b	Pumped-storage units at bus b
D_b	Loads at bus b
$L_b{}^{in}(L_b{}^{out})$	Lines to (from) bus b

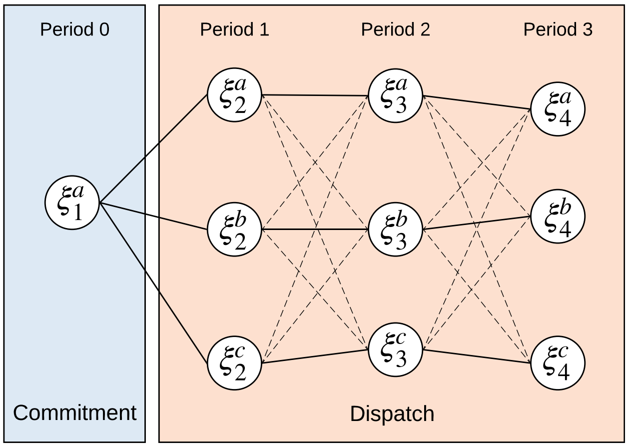
Table 3. Decision Variables

Notation	Description
$x_g{}^t$	Status (on/off) of thermal unit g at period t
$u_g{}^t$ $(v_g{}^t)$	Start-up (shut-down) decision of thermal unit g at period t
$p_g{}^t$	Generation of thermal unit g at period t
$g_w{}^t$ $(c_w{}^t)$	Generation (curtailment) of wind unit w at period t
$pc_p{}^t$ $(pd_p{}^t)$	Charge (discharge) of pumped-storage unit at period t
$s_p{}^t$	Stored energy in pumped-storage unit p at the end of period t
$\theta_b{}^t$	Phase (voltage) angle in bus b at period t
$f_l{}^t$	Flow in line l at period t
$gs_b{}^t$ $(ls_b{}^t)$	Exceeding generation (load shedding) in bus b at period t

more recently, Xiong et al. (2016) proposed a distributionally robust model where wind uncertainty is captured by a family of distributions (ambiguity set) and the worst-distribution case is optimized. Conversely, Ruiz et al. (2009) compared the stochastic and reserve methods and proposed a combined approach to deal with generation and load uncertainties, and Morales-España et al. (2018) compared the robust and the stochastic formulations and proposed a unified robust-stochastic formulation. All these models considered a two-stage formulation of the problem, as is usual in the UC literature. In what follows, we focus on stochastic formulations.

In the two-stage stochastic Unit Commitment (SUC) problem, it is assumed that uncertainty for the next 24 hours is revealed at the beginning of the day but many scenarios are considered simultaneously, with their respective probabilities. The first stage includes the commitment decisions (made without knowing which scenario will occur), and the second stage represents the dispatch decisions under each scenario (Figure 1). The latter are therefore decided under perfect foresight assumption in each scenario, which may be considered too optimistic to represent realistic system operations. This worsens as the coupling between interperiod decisions increases, as is the case when including energy storage facilities. In this situation, given a fixed wind and demand scenario, the energy storage capacity can be used to satisfy peak demand with stored energy to an unrealistic degree. The SUC problem may be formulated as a multistage stochastic optimization problem to overcome this difficulty. This, however, causes a significant increase in the computational burden.

Figure 1. (Color online) Stochastic Unit Commitment Decisions Structure



Notes. Each path represents a different scenario. Two-stage formulation only considers solid-line paths. Multistage formulation considers every possible combination of paths.

Shiina and Birge (2004) proposed an algorithm based on column generation to decompose a multistage SUC into single-generator subproblems; to limit the computational burden, they consider stages comprising several hours each. Related to this difficulty, Uçkun et al. (2016) proposed a model that sits in between the two-stage and the multistage formulations, putting the scenarios into buckets to reduce the problem's size and solve in extensive form. The work by Zou et al. (2018) proposed an interesting new decomposition algorithm based on stochastic dual dynamic integer programming (SDDiP) to solve a full multistage SUC problem. In the previously mentioned multistage settings, commitment decisions are made dynamically at each period as uncertainty is revealed. Although being attractive from an optimal operation perspective, these schemes may be impractical so far due to limitations in the implementation process since, currently, the optimization of commitment decisions is made one day ahead.

Hence, part of our goal is to formulate a model where commitments are decided one day ahead (as currently done by system operators in practice) while incorporating dynamic dispatch decisions to avoid misrepresentation due to the perfect foresight assumption (present, although to a different extent, in both the two-stage and deterministic formulations). We refer to this particular scheme of multistage SUC as the MSUC problem. We point out that we solve large instances of MSUC on real power systems using the SDDP method, described next.

2.2. SDDP in Short-Term Operations

Pereira (1989) proposed Stochastic Dual Dynamic Programming (SDDP) to solve the hydro-thermal scheduling problem: a large-scale multistage stochastic linear program. Since then, the method has been extensively studied (as a cross-discipline technique) and many variants proposed.

Recently, Papavasiliou et al. (2017) and Zéphyr and Anderson (2018) implemented SDDP to solve the real-time dispatch problem under wind uncertainty in the presence of energy storage facilities, demonstrating its capability to address short-term operational problems. We refer the reader to Guo et al. (2022) for a review on recent SDDP applications with energy storage and renewable integration. Although these formulations consider fixed commitment decisions, SDDP readily admits the inclusion of integer variables in the first stage. Therefore, it becomes natural to use SDDP to solve MSUC. This approach would solve a problem that decides the commitment of generating units in the first stage with dynamic dispatch decisions for the next 24 hours, avoiding the perfect foresight assumption.

The SDDP algorithm solves the multistage linear program:

$$(P)\ \min_{A_1x_1=b_1} c_1x_1 + \mathbb{E}_{\xi_2|\xi_1}\left[\min_{B_2x_1+A_2x_2=b_2} c_2x_2 + \mathbb{E}_{\xi_3|\xi_2}\left[\dots + \mathbb{E}_{\xi_T|\xi_{T-1}}\left[\min_{B_Tx_{T-1}+A_Tx_T=b_T} c_Tx_T\right]\right]\right], \tag{1}$$

where $\xi_t = (c_t, A_t, B_t, b_t)$, $t = 2, \dots, T$ represents the stochastic data process, containing discrete random vectors and matrices, with $\xi_1 = (c_1, A_1, b_1)$ being deterministic. In practice, B_t often has many columns of zeros; hence only a portion of x_{t-1} actually affects the inner optimization problems. This portion of x_{t-1} is commonly referred to as *state variables*; they carry all the necessary information to determine the initial state at stage t. Typically, it is assumed that every stage subproblem is always feasible for any possible value of x_{t-1} (known as the *relatively complete recourse* condition); otherwise, slack variables are introduced and penalized.

Notice the underlying scenario tree structure of the problem (P). Part of the efficiency of the SDDP method relies on having a fixed set of ξ_t samples at each stage and generating full-horizon scenarios as different combinations of them (this corresponds to a recombining scenario tree). In case ξ_t is continuous by nature (which is usual, as in our study), it must be discretized (e.g., by sampling or optimal quantization). In Section 4.1, we describe how to assess the quality of such approximations.

Problem (P) may be decomposed by stages. Then, for $t = T, \dots, 1$

$$Q_t(x_{t-1}, \xi_t) = \min_{B_tx_{t-1}+A_tx_t=b_t} c_tx_t + \mathcal{Q}_{t+1}(x_t, \xi_t), \tag{2}$$

where

$$\mathcal{Q}_{t+1}(x_t, \xi_t) = \begin{cases} \mathbb{E}_{\xi_{t+1}|\xi_t}\left[Q_{t+1}(x_t, \xi_{t+1})\right], & t = T-1, \dots, 1 \\ 0, & t = T. \end{cases} \tag{3}$$

The latter is known as the future value function. It accounts for the future effect of leaving the system in state x_t having observed ξ_t. It can be shown that the future value function is convex in x_t given ξ_t, which is critical for the algorithm to behave correctly. In SDDP, the future value function at each stage is progressively outer-approximated through Benders cuts. Each iteration comprises a forward pass, where areas of interest are determined to refine the approximation, and a backward pass, where the respective Benders cuts are calculated.

If ξ_t follows a stagewise-independent random process (as in the classic-SDDP method), it may be dropped as an argument of $\mathcal{Q}_{t+1}$. Therefore, only one set of cuts needs to be stored at each stage; otherwise, a different set of cuts must be stored under each possible value of ξ_t, because the probabilities involved in the cuts computation are dependent on it (as in the Markov-SDDP method). Other than this, both the classic- and Markov-SDDP methods are based on same principles to compute cuts. For details on the algorithm, please refer to (Philpott and De Matos 2012, Shapiro et al. 2013, Löhndorf and Shapiro 2019).

Should be highlighted that the stagewise-independent case can accommodate some forms of stagewise dependency by a reformulation of the original problem, in which the random parameters are assumed to follow either an additive or multiplicative dependency model with a stagewise-independent noise term, and previous stage values are added as state variables (Infanger and Morton 1996, De Queiroz and Morton 2013). However, these conditions may still be restrictive in some situations. For example, if random parameters are better represented by also including some nonlinear transformation.

3. Multistage UC Model with Temporal Correlations

In this section, we outline the proposed formulation of MSUC. Its key features are as follows:

1. Inclusion of all commitment decisions in the first stage under a multistage setting. To the best of our knowledge, this has not been done before. It is common to all the multistage stochastic UC models that we have seen that the commitment decisions are made dynamically as uncertainty reveals. Although these schemes are attractive from an optimal operation perspective, they may be impractical due to the current two-settlement process, where the optimization of commitment decisions are made day ahead, in advance of real-time operation.

2. Explicit incorporation of temporal correlations in an SDDP setting. Contrary to reformulation techniques, this admits a broad range of stagewise-dependency models. For example, we consider a sigmoid transformation of an autoregressive process to model wind power profiles.

3.1. Formulation of MSUC

We start by noticing that the nested structure of the problem (P) requires the state information to be passed from one stage to the next, and so on. This information includes the generation of thermal units at the previous stage (to define ramp rate limit constraints), the level of storage units (to declare energy inventory equations), and the committed schedule of thermal units decided in the first stage. Therefore, the entire schedule must be passed across stages, even if part of it is not needed until a later stage. This enlarges the state-space, which is the main drawback of our model; however, in Section 5.1, we will see a technique to deal with this difficulty.

All variables referenced to a time index less than 1 correspond to initial conditions and are assumed to be known.

3.1.1. Master Problem (Period 0). The master problem decides the commitment schedule of generating units (i.e., on/off statuses of power units). It minimizes the start-up and shut-down costs, considering the future cost of operating the system under the decided schedule. It can be written as

$$\min \sum_{g\in G}\sum_{t\in T}({u_g}^t {c_g}^{SU} + {v_g}^t {c_g}^{SD}) + \mathcal{Q}_2(\boldsymbol{x}_1) \tag{4a}$$

$$\text{s.t.} \quad {x_g}^t - {x_g}^{t-1} = {u_g}^t - {v_g}^t, \quad \forall g\in G,\ \forall t\in T; \tag{4b}$$

$$\sum_{d=0}^{UT_g-1} {u_g}^{t-d} \le {x_g}^t, \quad \forall g\in G,\ \forall t\in T; \tag{4c}$$

$$\sum_{d=0}^{DT_g-1} {v_g}^{t-d} \le 1 - {x_g}^t, \quad \forall g\in G,\ \forall t\in T; \tag{4d}$$

$${x_g}^t \in \{0,1\},\ {u_g}^t, {v_g}^t \ge 0, \quad \forall g\in G,\ \forall t\in T; \tag{4e}$$

where $\boldsymbol{x}_1 = (\{{x_g}^{t'}\}_{g\in G, t'\in T})$ denotes the state variables vector. Constraints (4b) link the generators' on/off status with the start-up and shut-down decisions. Constraints (4c) and (4d) enforce the minimum up and downtime of generators. The nature of variables is specified in (4e).

3.1.2. Subproblems (Periods 1 to 24). The system operation is represented by several consecutive subproblems, each comprising a one-hour frame, where demand is satisfied at minimum cost, considering the cost of subsequent subproblems. Generation cost is a quadratic function that we approximate piecewise-linearly but only activates when the respective commitment variable does. The subproblem at stage t is stated as

$$\min \sum_{i\in I} {w_{gi}}^t c_{gi}\Delta t + \sum_{b\in B}({C_p}^{ls} {ls_b}^t + {C_p}^{gs} {gs_b}^t)\Delta t + \mathcal{Q}_{t+1}(\boldsymbol{x_t}, \boldsymbol{\xi_t}) \tag{5a}$$

$$\text{s.t.} \quad {p_g}^t = \sum_{i\in I} w_{gi} {p_{gi}}^t, \quad \forall g\in G; \tag{5b}$$

$$\sum_{i\in I} {w_{gi}}^t = {x_g}^t, \quad \forall g\in G; \tag{5c}$$

$${x_g}^t {P_g}^{min} \le {p_g}^t \le {x_g}^t {P_g}^{max}, \quad \forall g\in G; \tag{5d}$$

$${p_g}^t - {p_g}^{t-1} \le {x_g}^{t-1} {R_g}^{up} + (1 - {x_g}^{t-1}){R_g}^{SU}, \quad \forall g\in G; \tag{5e}$$

$${p_g}^{t-1} - {p_g}^t \le {x_g}^t {R_g}^{down} + (1 - {x_g}^t){R_g}^{SD}, \quad \forall g\in G; \tag{5f}$$

$${g_w}^t + {c_w}^t = {I_w}^t, \quad \forall w\in W; \tag{5g}$$

$${s_p}^t = {s_p}^{t-1} + ({\eta_p}^{chg} {pc_p}^t - {pd_p}^t/{\eta_p}^{disc})\Delta t, \quad \forall p\in P; \tag{5h}$$

$${f_l}^t = B_l({\theta_{eb_l}}^t - {\theta_{sb_l}}^t), \quad \forall l\in L; \tag{5i}$$

$${\theta_{ref}}^t = 0; \tag{5j}$$

$${ls_b}^t \le \sum_{d\in D_b} {d_d}^t, \quad \forall b\in B; \tag{5k}$$

$${gs_b}^t \le \sum_{g\in G_b} {p_g}^t, \quad \forall b\in B; \tag{5l}$$

$$\sum_{g\in G_b} {p_g}^t + \sum_{w\in W_b} {g_w}^t + \sum_{p\in P_b} {pd_p}^t + \sum_{l\in {L_b}^{in}} {f_l}^t + {ls_b}^t = \sum_{d\in D_b} {d_d}^t + \sum_{l\in {L_b}^{out}} {f_l}^t + \sum_{p\in P_b} {pc_p}^t + {gs_b}^t, \quad \forall b\in B; \tag{5m}$$

$${w_{gi}}^t \ge 0, \quad \forall g\in G, i\in I; \tag{5n}$$

$$-F_l \le f_l \le F_l, \quad \forall l\in L; \tag{5o}$$

$$0 \le {pc_p}^t \le {Cap_p}^{chg}, \quad \forall p\in P; \tag{5p}$$

$$0 \le {pd_p}^t \le {Cap_p}^{disc}, \quad \forall p\in P; \tag{5q}$$

$${S_p}^{min} \le {s_p}^t \le {S_p}^{max}, \quad \forall p\in P; \tag{5r}$$

where $\boldsymbol{x_t} = (\{{s_p}^t\}_{p\in P}, \{{p_g}^t\}_{g\in G}, \{{x_g}^{t'}\}_{g\in G, t'\ge t})$ denotes the state information passed to the next-stage subproblem, and $\boldsymbol{\xi_t} = \{{I_w}^t\}_{w\in W}$ denotes the random information revealed to the subproblem at stage t. Constraints (5b) and (5c) define the piecewise-linear approximation of the generation cost function. Constraints (5d) specify the technical minimum and maximum power capacity limits for thermal generation, whereas ramping constraints are enforced by (5e) and (5f). Constraints (5g) specify the portions of wind power that are generated and curtailed. Energy inventory equations for storage units are specified in (5h). Linearized power flow equations (i.e., DC power flow equations) are represented by (5i) and (5j). Constraints (5k) and (5l) represent bounds on the slack variables. Demand balance equations are specified in (5m). Bounds on variables are represented by (5n)–(5r) constraints.

Notice that ${I_w}^t$ is a random parameter that represents the available energy due to wind and varies according to the scenario tree node being solved. Here, we explain how to build this tree.

3.2. Wind Uncertainty Model and Scenario Tree Sampling

In addition to the structure of the optimization subproblems (and master problem), we also have to describe the wind scenarios over which they are solved. This is encapsulated by a scenario tree that is generated before the start of the SDDP algorithm and according to the following wind uncertainty model.

Let $\boldsymbol{\xi}_t \in (0,1)^d$ be a random vector of normalized wind power at d wind units. We define $\boldsymbol{\xi}_t = S(\hat{\boldsymbol{\xi}}_t)$, where $S(\cdot): \mathbb{R}^d \rightarrow (0,1)^d$ is the component-wise sigmoid function, and we assume that the z-score of $\hat{\boldsymbol{\xi}}_t$ follows a vectorial AR(1) model with Gaussian noise, calibrated from data. Specifically, for each component we calibrate a separate AR(1), and then we fit a multivariate normal distribution to the noise terms to capture the spatial correlations. This way, the autoregressive component introduces the temporal correlations, whereas by jointly modeling the wind variables at different units as a random vector we capture the spatial correlations.

The autoregressive component, if alone, could be reduced to the stagewise-independent case through a reformulation of the problem, and be solved by classic SDDP. However, this is not the case when the nonlinear (sigmoid) transformation is included, which ensures the wind profiles do not exceed the maximum capacity of the unit, thus making the model more realistic. Therefore, we rely on the Markov-SDDP method to introduce the described uncertainty model.

Under this setting, we can sample as many wind scenarios as desired for the next 24 hours (we call these *base scenarios*), which we then use to build a recombining scenario tree as depicted in Figure 2. However, to account for the correct temporal correlations, transition probabilities must be carefully chosen; we rely on importance sampling (IS) for this task (Glynn and Iglehart 1989, Tokdar and Kass 2010).

In IS, samples from a given distribution are appropriately weighted to approximate another distribution of interest. In our case, the resulting set of wind samples at a given stage follows the marginal distribution at that stage, while the scenario tree is intended to approximate the conditional distribution. This way, according to the IS approach, transition probabilities may be calculated as

$$p\left(\boldsymbol{\xi}_t{}^j \,|\, \boldsymbol{\xi}_{t-1}{}^i\right) = \frac{w_t{}^{ij}}{\sum_k w_t{}^{ik}}, \tag{6a}$$

$$w_t{}^{ij} = \frac{1}{|J|} \frac{g(\boldsymbol{\xi}_t{}^j \,|\, \boldsymbol{\xi}_{t-1}{}^i)}{f(\boldsymbol{\xi}_t{}^j)}, \tag{6b}$$

where g and f are the conditional and marginal density functions of $\boldsymbol{\xi}_t$, respectively, and i, j are used to index samples. The previous definition actually corresponds to the normalized IS weights, where the term $w_t{}^{ij}$ corresponds to the ordinary IS weights. This introduces some bias in the approximation (although it converges asymptotically to zero) compared with ordinary IS, but the variance is generally reduced. Besides, they may be directly interpreted as probabilities.

A scenario tree with no temporal correlations structure may be constructed assuming that $\hat{\boldsymbol{\xi}}_t$ follows a pure noise process and considering equiprobable transitions. This will be useful in our experiments, as we are interested in assessing the value of modeling temporal correlations.

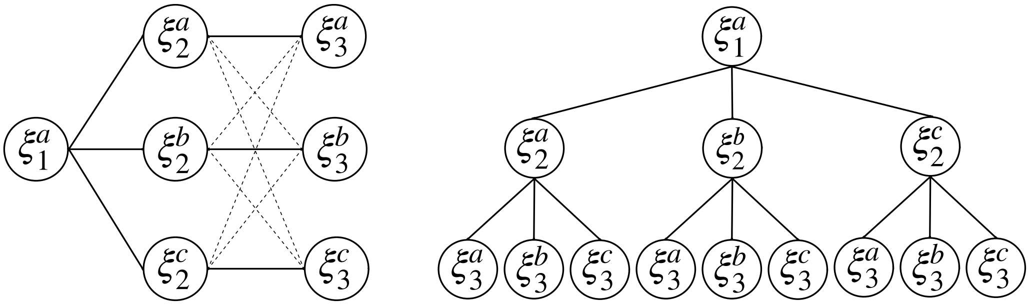
4. Experimental Settings

4.1. Repetition Analysis to Approximate the True Stochastic Problem

In reality, the wind uncertainty is a continuous random variable; however, for tractability, we formulate the MSUC problem on a discrete scenario tree. We refer to the MSUC-like problem with continuous random variables as the true problem and the (discretized) MSUC as the approximate problem.

Intuitively, a coarse-discretized uncertainty yields a smaller scenario tree that is easier to solve; however, its solution may behave poorly when evaluated under different scenarios. Conversely, a fine discretization would

Figure 2. Recombining Scenario Tree



Notes. (Left) Base scenarios. (Right) Recombined scenarios.

yield a good representation of uncertainty but would be impractical to solve. Therefore, it is necessary to measure the quality of the discrete approximation.

Under some mathematical assumptions, it is possible to derive statistical bounds on the true problem from the solution of the approximate problem through sample average approximation (SAA) (Shapiro 2003). To do so, we must repeatedly solve the MSUC problem under different scenario trees (sampled from the continuous distribution before each run, following the procedure from Section 3.2) and compute bounds from the results.

The upper bound from SAA is obtained from any feasible solution to the MSUC because a solution for a given scenario tree yields an implementable policy. The expected cost of this solution can be estimated by sampling a great number of scenarios from the continuous distribution. This "out-of-sample value" is the upper end of the confidence interval for the mean cost over simulations. The upper bound on the true problem is the minimum out-of-sample value among the different scenario tree solutions.

The SAA lower bound is obtained by virtue of Jensen's inequality. This result implies that the mean of the optimal values of the MSUC yields a lower bound on the optimal value of the true problem. The same holds if the approximate problems are not solved to optimality, but a lower bound is known (as is the case in SDDP if we take the master problem value at any iteration). Because we are solving a limited number of approximations (or repetitions), the lower bound on the true problem is obtained as the lower end of the confidence interval for the mean across repetitions.

The stochastic gap measures the quality of the solutions for a given discretization of the scenario trees. It provides a bound with respect to the optimal solution of the true problem, removing the bias induced by any specific tree chosen for the optimization procedure. The stochastic gap is defined as the relative difference between the upper and lower bounds on the true problem.

As the IS procedure explained in Section 3.2 provides an (asymptotically) unbiased estimator, the derived approximate problem is also valid to compute the SAA statistical bounds; interested readers can find the details in Cordera et al. (2021).

4.2. Performance Metric for Unit Commitment Solutions

We are interested in comparing different SUC formulations (e.g., multistage versus two-stage). Therefore, we need a common metric to evaluate the quality of the commitment solutions each problem formulation obtains. Given a commitment solution, we evaluate its performance as the expected objective function value of the multistage formulation with temporal correlations, with the commitment decisions fixed. We construct SAA bounds for this problem. We choose this formulation as it more realistically represents the time horizon in which dispatch decisions are made, and therefore better approximates real operational costs.

Therefore, our experiments consider two phases: optimization and evaluation. The former refers to the phase where formulations are solved to produce commitment solutions, whereas, in the latter, we assess (under a common metric) the quality of the solutions found. In the evaluation phase, we use the same scenario trees and out-of-sample scenarios to find and simulate, respectively, the dispatch policy associated with the different commitment solutions.

4.3. IEEE 118-Bus System

The IEEE 118-bus System comprises 54 thermal units, 91 load nodes, and 186 branches, arranged in three zones. Additionally, we include three wind units (800 MW of installed capacity each) and three pumped-storage units. We locate these additional units at buses 17, 59, and 69 (one wind-storage pair per bus). The pumped-storage units consider a maximum power capacity of 300 MW for both charge and discharge, a round-trip efficiency of 0.75, and a maximum energy storage capacity of 1,800 MWh. To avoid an oversized system due to these additions, we reduce the capacity of the thermal units to 75% of their original values. The peak demand of the system is 6,600 MW, and its total installed capacity is 9,248.5 MW. We consider a high penalty cost of 5,000 \$/MWh for both exceeding generation and load shedding. We also modify the original ramp rate limits and generation costs of thermal units. The characterization by fuel type is shown in Table 4.

Following the global trend to decarbonize power systems, we consider a coal-reduced scenario to drive sensitivity analysis on the proposed approach. We apply a

Table 4. Characterization of Thermal Units (Mean Values)

Fuel type	Installed capacity (MW)	Units	Maximum power (MW)	Minimum power (MW)	Minimum up/down time (h)	Ramp rate (MW/h) (relative to maximum power)	Marginal cost (\$/MWh)
Coal	5,411.3	33	164.0	58.6	6.6	30%	39.2
Gas	266.3	11	24.2	6.8	1.0	62%	78.9
Oil	271.0	10	27.1	12.4	1.2	100%	150.0

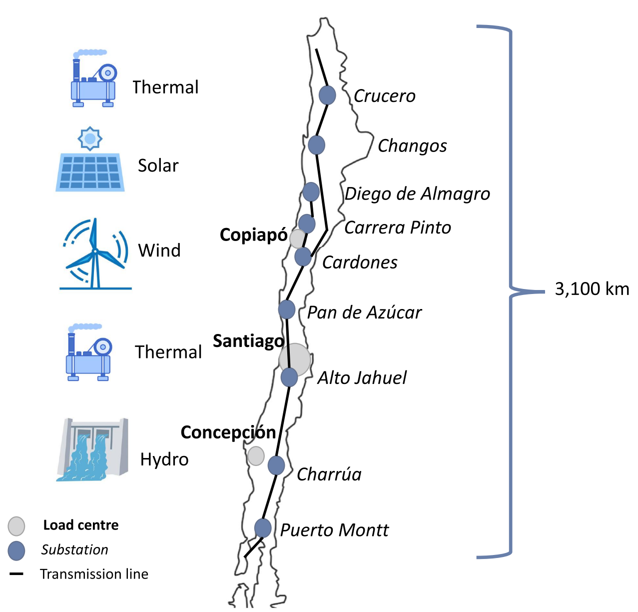
20% reduction on the original maximum power of coal units, and again we limit all the thermal units to 75% of their (resulting) generation range.

System data are obtained from a public repository of the Illinois Institute of Technology (2003). The wind power stochastic process is calibrated from wind data by Pinson (2013).

4.4. Chilean System: A Real-World Application

To test the scalability and materiality of our approach on a real-world application, we consider the Chilean power system by 2021 (current) and 2030. In 2021, the system comprises 227 buses, 332 branches, 137 load nodes, 166 thermal generators (11 GW), 20 wind units (2.3 GW), 52 solar units (3.3 GW), and 28 hydropower generators (6.8 GW), 6 of them with dam regulation (4.9 GW). The system installed capacity is 23.5 GW and the peak demand in the day studied is 11 GW. We consider a high penalty cost of 5,000 $/MWh for exceeding generation and 1,000 $/MWh for load shedding. We assume wind power as the only source of uncertainty, as its predictability is significantly lower than solar power and demand. In 2030, and following government's projections (Ministerio de Energía 2021), we assume that demand will increase by 25%, coal capacity will be reduced to 3.1 GW, and solar and wind capacity will increase to 9.8 and 8.3 GW, respectively. We also assume the installation of new transmission lines in accordance with government's plan. See Figure 3 for an stylized diagram of the system. The associated data can be download from Cordera et al. (2023).

Figure 3. (Color online) Stylized Representation of the Chilean Electricity System, Indicating Key Pieces of the Transmission Network and the Location of Main Load Centers and Generation Resources



Because of the important presence of hydro resources, we carry out our analyses under two hydro scenarios: normal and dry. We disregard wet hydro scenarios as their excess of flexibility from hydro resources makes the variability of wind generation irrelevant in deciding the unit commitment problem for the next day and does not provide an interesting case study as those under drier scenarios. Dry hydro scenarios seek to model scarcity conditions in the electricity market, which, in reality, may not be triggered necessarily by a generalized lack of hydro resources but by increased values of other water uses that compete against energy production (e.g., recreational, environmental, agriculture). Going forward, this will become progressively more important due to the increased socio-environmental awareness by policymakers and regulators.

5. Computational Results

5.1. SDDP Configuration: Convergence and Regularization

As general SDDP settings, we sample 30 base scenarios to derive a recombining scenario tree (which is generated prior to running the algorithm), we implement a single-cut approach and perform only one simulation at each forward pass. As the stopping criteria, we consider a maximum of 400 iterations or six hours of execution time or a 0.1% convergence gap. To evaluate convergence, we update the upper bound estimation every 50 iterations, based on 200 simulations (to reduce variance in comparing the upper bounds, we use the same 200 scenarios each time, sampled from the recombining scenario tree at the beginning). At the end of the algorithm, we choose the minimum upper bound policy (which is not necessarily the one from the last evaluation). Finally, the chosen policy enters as a candidate for best policy in the SAA procedure.

The iterative nature of SDDP causes high variability of the solution objective value after each iteration. In high-dimensional spaces, this may turn into a chaotic search with extremely slow convergence (Asamov and Powell 2018). Indeed, our preliminary tests presented this behavior when including commitment decisions at the master problem, which increases the state-space dimension because the complete schedule must be passed across stages. Regularization techniques may significantly fasten SDDP convergence in high-dimensional settings, as shown by Asamov and Powell (2018), who developed a quadratic regularization approach with proven convergence to an optimal policy after a finite number of iterations. They introduced regularization as a vanishing (as iterations progress) term in the objective function that penalizes deviations of the state variables with respect to their values in the last forward pass (we call this the *reference value*). Its purpose is to steer the solution toward a known region of the future value function. The regularization term is

$$\frac{1}{2}\rho^k(\boldsymbol{x}_t - \underline{\boldsymbol{x}}_t{}^k)^T \boldsymbol{R}_t(\boldsymbol{x}_t - \underline{\boldsymbol{x}}_t{}^k), \tag{7}$$

where $\rho \in [0,1)$ is the decaying rate of the regularization, which is raised to the power of k (the iteration index), $\underline{\boldsymbol{x}}_t{}^k$ is the kth iteration reference value, and $\boldsymbol{R}_t$ is a positive semidefinite matrix used to address any scaling concerns across the state vector components.

As SDDP relies on sampling, the scenarios at two consecutive iterations may significantly differ from each other, and therefore deviations may actually be desired. To soften the impact of these extreme shifts, we instead define the reference value as an average of the previous iterations. Specifically, we consider an exponential moving average (which places a greater weight on the most recent values):

$$\underline{\boldsymbol{x}}_t{}^{k+1} = (1-\alpha)\underline{\boldsymbol{x}}_t{}^k + \alpha \boldsymbol{x}_t{}^k, \tag{8a}$$

$$\underline{\boldsymbol{x}}_t{}^2 = \boldsymbol{x}_t{}^1, \tag{8b}$$

where $\alpha \in (0,1]$ is a coefficient that represents the degree of weighting decrease, and $\boldsymbol{x}_t{}^k$ is the kth iteration solution value. Another difference in our regularization approach is to use the absolute-value distance instead of the Euclidean distance; the sole purpose of this is to keep things linear. Thus, our regularization term is

$$\rho^k \boldsymbol{r}_t{}^T |\boldsymbol{x}_t - \underline{\boldsymbol{x}}_t{}^k|, \tag{9}$$

where $\boldsymbol{r}_t$ is a nonnegative vector used to scale components. The nature of this term is similar to that of Asamov and Powell (2018), as it vanishes as iterations progress; we only use a different norm.

To determine the value of $\boldsymbol{r}_t$, we solve a few iterations of the algorithm with no regularization and then choose the average coefficients of the Benders cuts for each state variable. The purpose of this procedure is simply to estimate a reasonable penalization scaling. Additionally, we consider $\rho = 0.99$ and $\alpha = 0.7$.

5.2. Results on the IEEE 118-Bus System

To assess the value of the proposed MSUC formulation, we compare the commitment solution's performance determined by different schemes of temporal correlation and time horizon representation. We solve the following cases:

- MS-Markov: MSUC with temporal correlations.
- MS-indep: MSUC with no temporal correlations.
- 2S-Markov: two-stage SUC with temporal correlations.
- 2S-indep: two-stage SUC with no temporal correlations.
- determ: deterministic UC with reserve requirements.

The MSUC model is described in Section 3.1, and temporal correlations are included by means of the transition probabilities determined by the procedure described in Section 3.2. Besides, the base scenarios uses to derive a multistage tree are used to build the respective two-stage counterpart, and the scenarios are equally weighted. The two-stage SUC is described in Appendix A.

To estimate the stochastic gap, the stochastic formulations (namely, two-stage and multistage) are solved considering five repetitions evaluated under 1,000 out-of-sample wind power scenarios generated from the corresponding temporal correlation scheme. For example, for MS-Markov, this means we generate five different recombining scenario trees whose probabilities account for the temporal dependency, we solve each of them with SDDP, and the resulting policies are evaluated under the same set of out-of-sample scenarios. The same for MS-indep, but the trees and out-of-sample scenarios are generated as if no temporal correlation exists.

The deterministic UC approach, determ, is of practical interest as it is a current standard in the power industry to determine the commitment schedule of generating units. We solve it using the mean wind power at each stage, and the reserve requirements at each zone are computed as the difference between the maximum and mean power generation of nonrenewable units (effectively, we assume that renewables cannot provide reserve services, although this may change in the future) under the MS-Markov policy. These requirements are somewhat idealized because they are determined based on the stochastic version of the problem, which may not be available in practice.

Table 5 shows the summarized results for the above models. In both the optimization and evaluation phases, the *best value* refers to the best solution value found for

Table 5. Summarized Results for the Optimization and Evaluation Phases

	Optimization		Evaluation		
Model	Best value (10^2 \$ per day)	Stochastic gap	Best value (10^2 \$ per day)	Stochastic gap	Relative performance
MS-Markov	27,169.91	3.46%	27,147.83	0.83%	0.00%
MS-indep	27,255.91	9.54%	27,535.93	1.96%	1.43%
2S-Markov	26,859.12	0.39%	27,739.26	3.82%	2.18%
2S-indep	26,811.00	0.16%	28,012.50	4.75%	3.19%
determ	26,815.26	—	27,882.13	4.39%	2.70%

the respective optimization problem, whereas the *stochastic gap* relates to the optimality guarantees (not shown for *determ* in the optimization phase because it is not a stochastic model). In the evaluation phase, the *relative performance* is with respect to the lowest *best value* policy; in this case, the one derived from the MS-Markov commitment solution. Best value is not a comparable performance metric in the optimization phase because the values refer to conceptually different optimization models.

In Table 5, the results of the evaluation phase show that it is more valuable to formulate UC in a multistage fashion rather than including temporal correlations, although including neither is even worse than the deterministic formulation with reserve requirements.

Ideally, one would like to solve all models to the same gap level to make clearer comparisons. However, it is not possible to determine the time and resources needed to do that in practice. In this sense, we think the difficulty to reach a specific gap (with limited resources) is inherent to a given model and is part of the comparison. Therefore, we make sure to use the same configurations (namely, the level of sampling and stopping criteria) any time we compare multistage problems solved by SDDP, which are the most time-consuming in our experiments. In general, the deterministic, two-stage and multistage runs take dramatically different computational times by nature, on the order of seconds, minutes and hours, respectively.

5.2.1. Commitment Results. Figure 4 shows the commitment solutions determined in the optimization phase. As expected, the perfect foresight assumption in both the two-stage and the deterministic formulations (albeit to a different extent) permits to efficiently accommodate generation across the day to avoid the use of more expensive thermal units. This is in opposition to the case of multistage formulations (this behavior is complemented with Figure 5, where we notice that multistage formulations commit higher amounts of thermal capacity). Conversely, we notice that the incorporation of temporal correlations (Markov formulations) makes the commitment solutions look more spread across the day compared with their uncorrelated counterparts. This may be explained by noting that extreme (lower, in particular) wind events are likely to persist in time when temporal correlations exist, and the system must deal with them more frequently than in the absence of correlations, where extreme events tend to cancel across time.

5.2.2. Operational Results. In this section, we show the solutions obtained by simulating the policies found in the evaluation phase. Results are averaged across 1,000 out-of-sample scenarios.

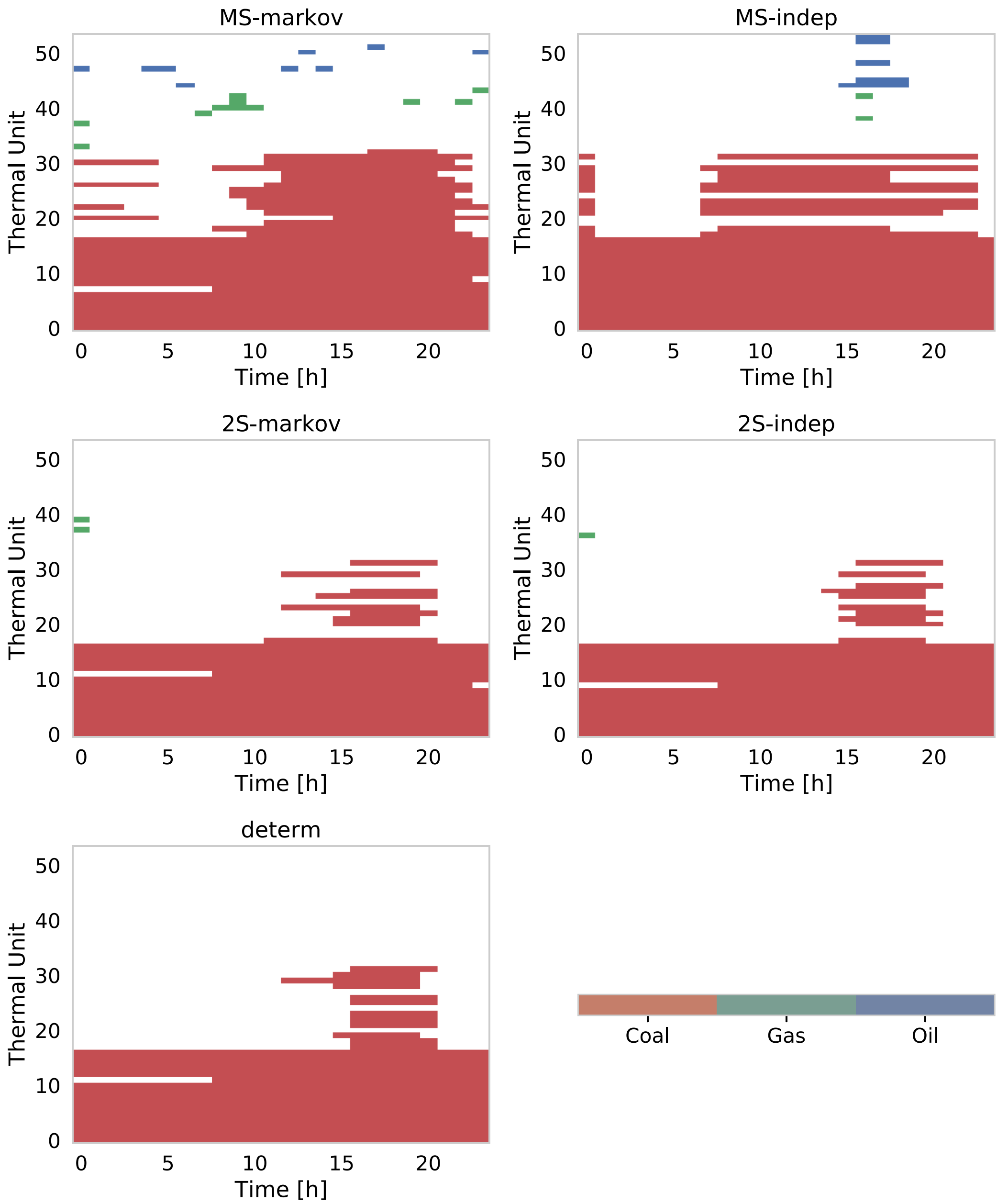
We verify that exceeding generation is zero for all cases. Conversely, Figure 6 shows that both the amount and frequency of unsatisfied demand events are higher for commitment solutions different from MS-Markov. This impacts the dual prices of the nodal balance constraints (i.e., nodal energy prices), whose values equal the penalty cost any time load shedding occurs. Figure 7 presents the evolution of these prices during the day and their variation across nodes; lower prices indicate that an increase in demand can be accommodated more efficiently.

With these energy prices, we can determine the demand payments (the amounts of money to be paid by customers for the energy supplied). Figure 8 compares these demand payments under different commitment solutions; results are consistent with the evaluation phase values shown in Table 5, in the sense that lower-cost solutions report lower demand payments. This illustrates that the inefficiencies of a lower-quality commitment solution directly affect customers in a negative fashion. Interestingly, although the two-stage and deterministic formulations commit fewer thermal units (and capacity) than the multistage formulations, the demand payments are higher. This means that a higher total income is distributed among fewer participants, whereas the opposite occurs in the multistage cases. Another interesting finding is that the most expensive solutions from the consumers' viewpoint are those delivering lower reliability levels. Thus, MS-Markov solution is the most efficient one from both the economic (Figure 8) and reliability (Figure 6) perspectives. These results demonstrate that system operators and regulators should be careful because different approaches lead to different economic and reliability performances and significantly different prices, which, in turn, may affect generators' incentives and send incorrect price signals to market participants, as pointed out by Sioshansi et al. (2008).

5.2.3. Coal-Reduced System. Table 6 shows the results of the experiments on the coal-reduced system. As before, the MS-Markov is the best solution. However, the difference with respect to the other stochastic formulations is reduced, whereas the determ solution is significantly worse. By reducing the coal capacity, the system is pushed to rely more on oil and gas units, which are more expensive but also faster. This may reduce the impact of wind variability as the system can respond more quickly to different contingencies, which explains the smaller differences between stochastic formulations. Conversely, both the reduction in system capacity and the consequent use of more expensive generation units may boost the participation of energy storage units (despite undesirable charge/discharge inefficiencies), which would more negatively affect the deterministic formulation as it is less prepared to deal with high net demand scenarios (demand minus the contribution from renewables).

Importantly, considering the global trend to decarbonize power systems, operators should be aware of these results since the deterministic model is the current standard in industry.

Figure 4. (Color online) Commitment Solutions (Optimization Phase)



Note. Thermal units are ordered according to increasing generation cost (e.g., unit 0 represents the cheapest unit).

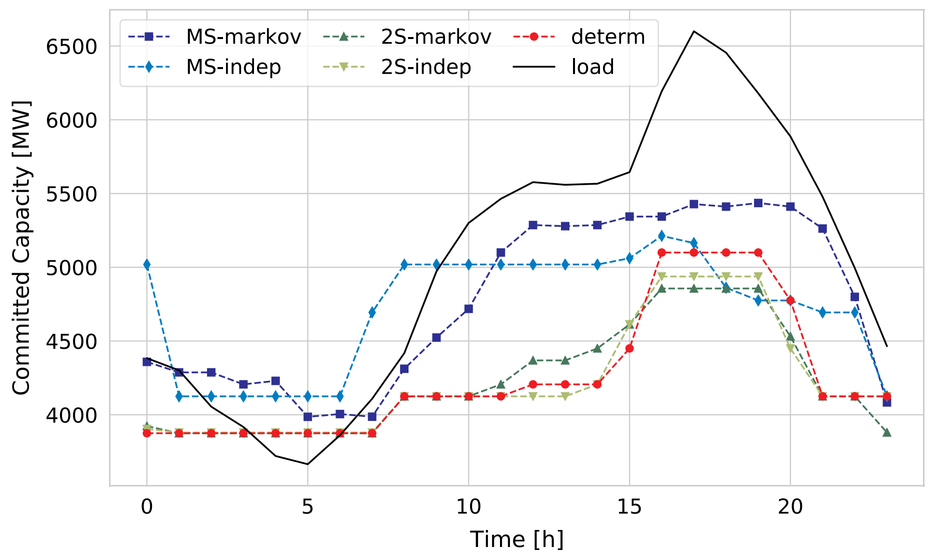
5.3. Results on the Chilean System

5.3.1. Current Practice. We compare the solution's performance of the MS-Markov, 2S-Markov, and determ approaches. To appropriately represent the current deterministic practice in Chile (Coordinador Eléctrico Nacional 2021), the latter, renamed as determ-desv2, is determined with reserve requirements calculated as two standard deviations of the wind forecast error at each hour ($2\sigma^t_{wind}$). Additionally, we also evaluate determ-desv3, the traditional $3\sigma^t_{wind}$ criterion (for a discussion on this subject, see Silva 2010) to study the performance of the Chilean approach if reserves were increased.

When solving the stochastic formulations (i.e., MS-Markov and 2S-Markov), the stochastic gap estimation relies on the same settings used in the IEEE 118-bus system experiments (i.e., number of repetitions, discretization level, and out-of-sample scenarios). Table 7 shows the stochastic gaps reached in the optimization phase. We notice that 2030 cases are more difficult to solve, but still a reasonable gap is reached.

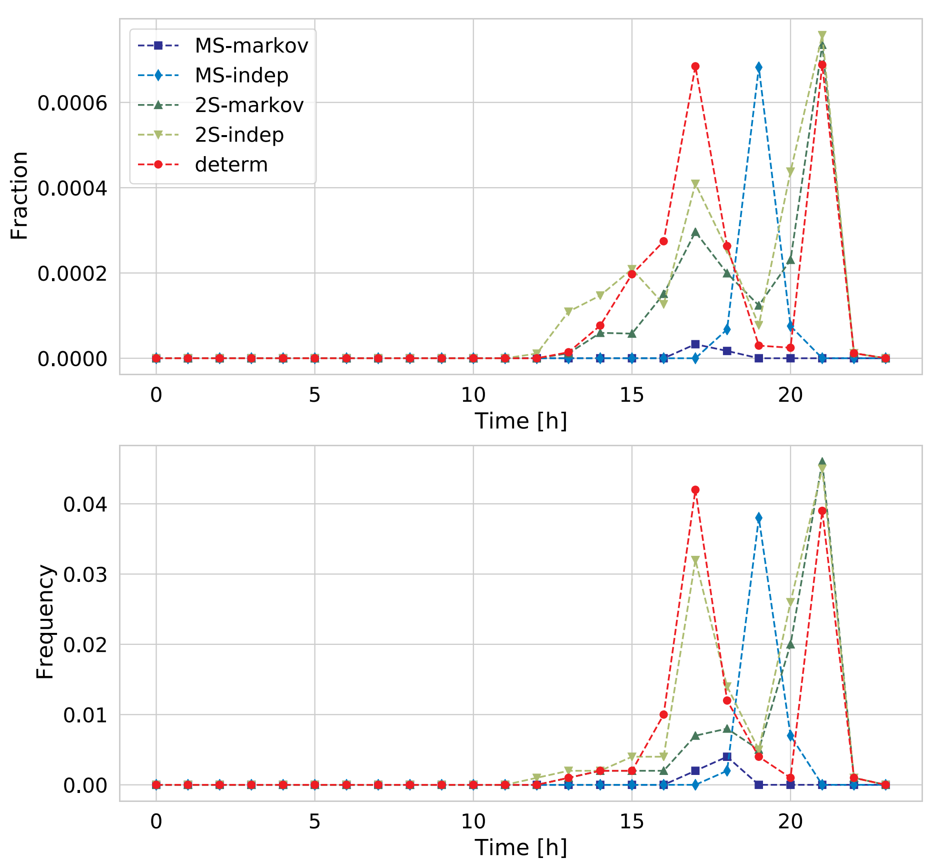
Figure 9 compares the expected costs of commitment solutions (including start-up, shut-down and generation costs) when evaluated over the same set of scenarios. We notice that the commitment solutions coming from

Figure 5. (Color online) Committed Thermal Capacity



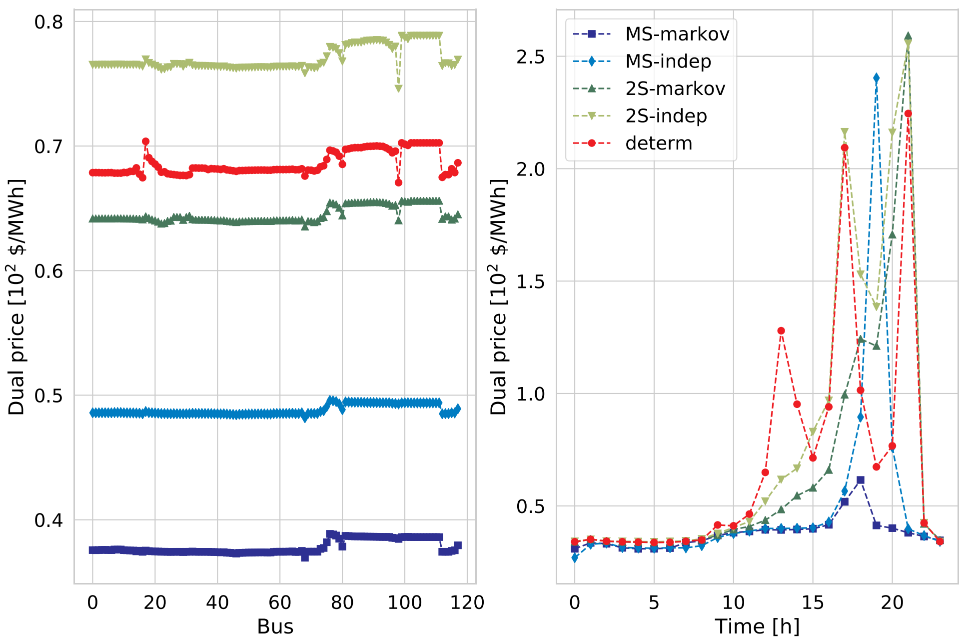
deterministic approaches have a slightly smaller expected cost than the stochastic ones in the current system. This, however, is reversed for the projected 2030 system, where stochastic approaches have a significantly smaller expected cost than the deterministic ones. Every model evaluated incurs in higher costs in the dry scenario.

Figure 6. (Color online) Load Shedding



Notes. (Top) Volume of unsatisfied demand (on average) as a proportion of the total demand. (Bottom) Number of scenarios where load shedding occurs as a proportion of the total number of scenarios.

Figure 7. (Color online) Dual Prices



Notes. (Left) Average dual price over the day at each bus. (Right) Average dual price over all buses at each hour.

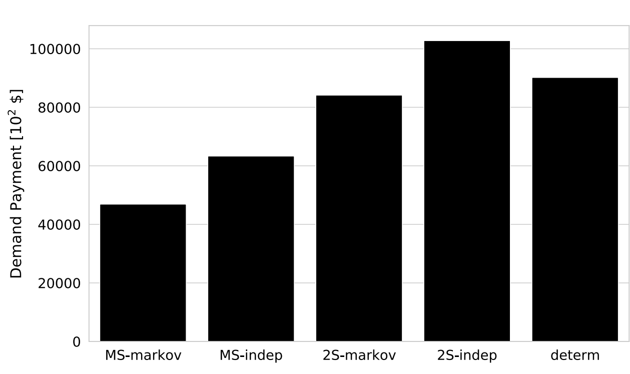
Regarding the small differences observed among the solutions in 2020 (about 0.1% of the total cost of the day), these can be explained due to the high flexibility levels of the current Chilean system (provided by ramp-flexible thermal units and important storage capacity of reservoir units). This flexibility allows to straightforwardly accommodate moderate wind variations even when uncertainty is not explicitly modeled (as in the deterministic models). Because the installed wind capacity is more significant in 2030 and so is the impact of its uncertainty, the system becomes less flexible to adapt to the higher wind uncertainty, leading to the larger cost differences among the UC decisions.

Regarding the better performance of deterministic models in 2020, this can be justified because stochastic approaches are harder to solve, becoming increasingly harder near optimality. Therefore, some efficiency can be lost compared with deterministic approaches when uncertainty is low, which is accentuated in the MS-Markov (over 2S-Markov) due to its inherent complexity.

As mentioned earlier, interestingly, the improvements of the proposed approach are more significant in the 2030 system under the dry scenario, which is justified due to the greater presence of uncertainty and the limited capability from hydropower plants to provide storage/flexibility services.

Finally, the $2\sigma_{wind}$ criterion used currently by the Chilean ISO tends to outperform the more widely used $3\sigma_{wind}$ criterion. This means that, for the case of Chile, the latter overestimates the amounts of economically justifiable reserves that are required to secure system operation. This also demonstrates the importance of adopting, in deterministic approaches, appropriate requirements of reserves that are both technically and economically justifiable. Next, we argue that appropriate reserve requirements to be considered in deterministic models can be found using stochastic approaches. This would be a straightforward manner to introduce stochastic UC models in practice.

Figure 8. Demand Payment Under Different Commitment Solutions



5.3.2. Straightforward Combination of Deterministic and Stochastic UC Models. The need for reserves to hedge system operation against risks originated by renewable sources depends on a number of factors. Importantly, the exact amounts of reserves, their allocation among power units, their locations in the network, and their dynamics are endogenously determined by MSUC, whereas, in deterministic approaches, these have to be determined exogenously. In this regard, we evaluate an additional deterministic approach to the previous

Table 6. Summarized Results for the Optimization and Evaluation Phases in the Coal-Reduced System

	Optimization		Evaluation		
Model	Best value (10^2 \$ per day)	Stochastic gap	Best value (10^2 \$ per day)	Stochastic gap	Relative performance
MS-Markov	28,775.92	5.06%	29,197.76	3.01%	0.00%
MS-indep	29,123.86	8.15%[a]	29,516.74	4.61%	1.09%
2S-Markov	28,301.87	1.72%	29,496.53	5.17%	1.02%
2S-indep	28,013.75	0.53%	29,556.68	5.21%	1.23%
determ	27,784.16	—	33,218.36	7.13%	13.77%

[a]In the optimization phase, we notice that one of the repetitions in the MS-indep case exhibited notoriously poor convergence. We recognize it as an outlier and exclude it from the stochastic gap calculation.

ones, determ-msuc, based on reserves calculated by the MS-Markov case (as explained before in the IEEE 118-bus system section).

In Figure 10, we compare the expected cost increase (relative to MS-Markov) associated with the commitment solutions coming from the previous determ-desv2 and determ-desv3 but also from the new determ-msuc. We notice that determ-msuc significantly outperforms the other reserve-based UC approaches in the 2030 system under the dry scenario. In the other conditions, where the system features enough flexibility to adapt to wind uncertainties, the determ-msuc approach remains comparable to the current practices. Notably, the improvement associated with determ-msuc in the 2030 dry scenario is related to the specific dynamics of the assigned reserve capacity, which is appropriately captured by the MSUC model.

Furthermore, we believe that the determ-msuc solution can be further improved by creating various zones or subsystems with different reserve requirements (determined by the MSUC model). However, this will require modifications in the ancillary services market to recognize different requirements per zone. Instead, the current system-wide reserve requirements used in the Chilean deterministic UC model can be straightforwardly improved by calculating them using the MSUC model as demonstrated here. This represents a reasonable first step toward the full application of stochastic UC models.

6. Conclusions

In systems with high uncertainty in energy availability, as those with increasing participation of renewables, current practice in determining the commitment schedule of generating units can lead to poor solutions. We propose a multistage SUC model under temporal correlations from renewable generation. Our model comprises a multistage setting that can be solved using a variant of the SDDP algorithm, including regularization techniques to accelerate convergence, which allows handling large-scale problems. We numerically demonstrate its capabilities to produce high-quality commitment solutions.

Experiments on the IEEE 118-bus system, in which both renewables and energy storage are considered, show that our model outperforms other stochastic models (e.g., two-stage) and the traditional deterministic formulation (with reserve requirements). In multistage settings, convergence is harder when temporal correlations are disregarded and produce poorer quality solutions. It is also shown that the deterministic formulation significantly deteriorates (relative to other stochastic formulations) when coal-units capacity is reduced, which is a key consideration going forward in the coming decarbonization era. A vital feature of the solution obtained by our model is that both economic and reliability performances are superior to the rest, meaning that our solution is the one with the lowest expected energy not supplied, being the most affordable to consumers too.

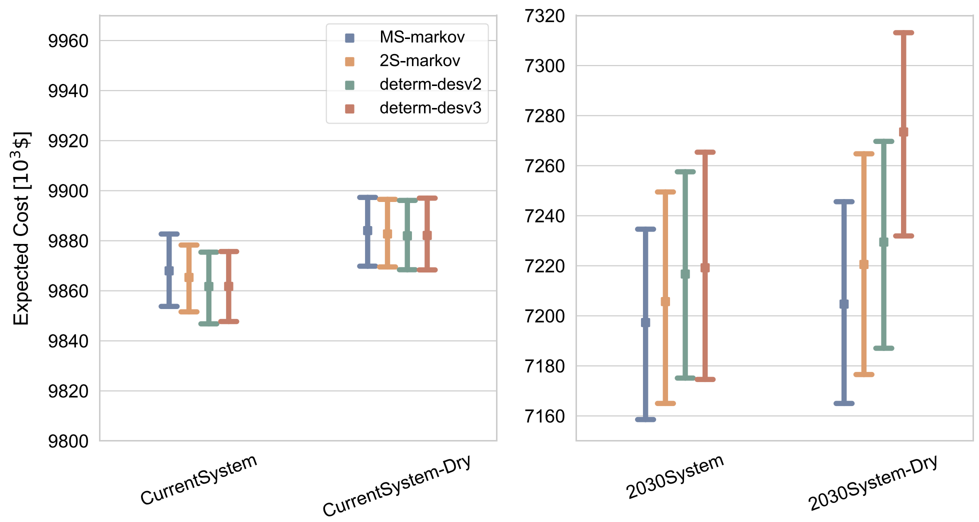
Results on the Chilean system show that the proposed methodology scales to solve real-world problems. We observe that, currently, the Chilean system features sufficient fast and flexible generation capacity, reducing the value of the stochastic solution. However, we also observe that, going forward, in situations with increased renewables' penetration and degraded generation flexibility (i.e., less hydro availability), the proposed methodology provides a more cost-effective unit commitment solution than the current practice in industry. Furthermore, we propose a deterministic UC strategy based on our MSUC approach that is akin to the current industry practice and outperforms it in situations without generation flexibility.

Apart from the previously mentioned conclusions, particular to our work, stochastic optimization features other general characteristics that are essential going

Table 7. Stochastic Gap for the Optimization Phase in the Chilean System

Model	MS-Markov	2S-Markov
CurrentSystem	1.10%	0.79%
CurrentSystem-Dry	0.97%	0.64%
2030System	3.23%	2.23%
2030System-Dry	3.16%	2.31%

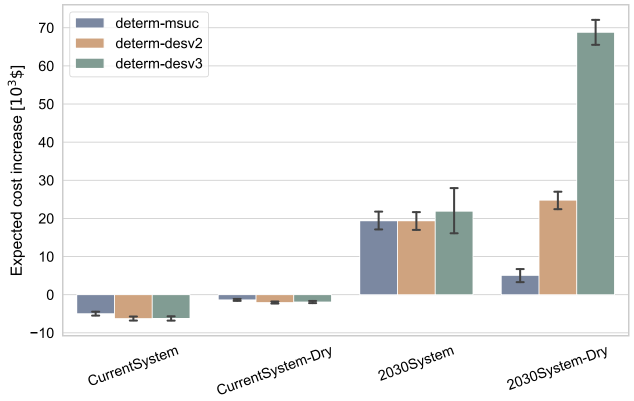
Figure 9. (Color online) Expected Cost (and 95% Confidence Intervals) of Commitment Solutions in the Chilean System



Notes. (Left) Current system. (Right) 2030 system.

forward. First, stochastic UC endogenously tests whether power can be delivered in real-time operations and, by doing so, identifies, in advance, the units that should be committed in the next day. This is a fundamental difference with the current deterministic approach applied by industry, where the allocation of the (exogenous) reserve requirements among generators does not recognize network congestion and other complexities that occur in real-time operations. Second, stochastic UC directly balances costs and risks in its objective function, which is fundamentally different from deterministic models that minimize costs only under an "average" scenario. The latter is significantly problematic in future power systems dominated by variable renewable generation and can lead to commitment plans that are not prepared to deal with actual realizations of uncertain parameters (e.g., renewable generation). Clearly, stochastic optimization shows significant advantages in operating future power systems, although complexity, tractability, and the computational burden associated are issues that must be appropriately addressed. In this context, we successfully demonstrate how to efficiently solve hard, realistic instances of MSUC with temporal correlations by using SDDP. Moreover, without our model, treatment of temporal correlations (which are fundamental to capture renewables uncertainty realistically) within SDDP will be significantly less efficient.

Figure 10. (Color online) Expected Cost Increase (and 95% Confidence Intervals) of Deterministic Commitment Solution, Relative to the MS-Markov Approach, in the Chilean System



Algorithmically, although the Benders cuts (used in our SDDP implementation) are tight, they are not necessarily the best. This is because they implicitly assume that the state variables are continuous, whereas, for example, commitment variables are binary. In this vein, future work should consider alternative cuts such as those presented in Zou et al. (2019). Specifically, Lagrangian cuts may improve convergence as they better approximate the future value function by incorporating the information related to the binary nature of some state variables.

Appendix A. SUC: Two-Stage Formulation

The two-stage formulation of SUC is closely related to MSUC (presented in Section 3.1). They both have the same master problem and share the same operational constraints in the subproblem but with a different time horizon representation. Although MSUC has 24 nested subproblems, each representing a 1-hour frame for the next 24 hours, the two-stage counterpart has a single (and bigger) subproblem accommodating all the constraints and objectives for the next 24 hours. This single subproblem makes the operational decisions of the first hour knowing the wind scenario in the last hour of the day, and therefore the problem assumes perfect foresight in each scenario. Since the difference is in the subproblem, here we only present the subproblem formulation:

$$\min \sum_{t\in T}\left(\sum_{i\in I} w_{gi}{}^t c_{gi} + \sum_{b\in B}(C_p{}^{ls} ls_b{}^t + C_p{}^{gs} gs_b{}^t)\right)\Delta t \quad \text{(A.1a)}$$

$$\text{s.t.}\quad p_g{}^t = \sum_{i\in I} w_{gi}{}^t p_{gi}{}^t, \quad \forall g\in G,\ \forall t\in T; \quad \text{(A.1b)}$$

$$\sum_{i\in I} w_{gi}{}^t = x_g{}^t, \quad \forall g\in G,\ \forall t\in T; \quad \text{(A.1c)}$$

$$x_g{}^t P_g{}^{min} \le p_g{}^t \le x_g{}^t P_g{}^{max}, \quad \forall g\in G,\ \forall t\in T; \quad \text{(A.1d)}$$

$$p_g{}^t - p_g{}^{t-1} \le x_g{}^{t-1} R_g{}^{up} + (1 - x_g{}^{t-1}) R_g{}^{SU}, \quad \forall g\in G,\ \forall t\in T; \quad \text{(A.1e)}$$

$$p_g{}^{t-1} - p_g{}^t \le x_g{}^t R_g{}^{down} + (1 - x_g{}^t) R_g{}^{SD}, \quad \forall g\in G,\ \forall t\in T; \quad \text{(A.1f)}$$

$$g_w{}^t + c_w{}^t = I_w{}^t, \quad \forall w\in W,\ \forall t\in T; \quad \text{(A.1g)}$$

$$s_p{}^t = s_p{}^{t-1} + (\eta_p{}^{chg} pc_p{}^t - pd_p{}^t/\eta_p{}^{disc})\Delta t, \quad \forall p\in P,\ \forall t\in T; \quad \text{(A.1h)}$$

$$f_l{}^t = B_l(\theta_{eb_l}{}^t - \theta_{sb_l}{}^t), \quad \forall l\in L,\ \forall t\in T; \quad \text{(A.1i)}$$

$$\theta_{ref}{}^t = 0, \ \forall t\in T; \quad \text{(A.1j)}$$

$$ls_b{}^t \le \sum_{d\in D_b} d_d{}^t, \quad \forall b\in B,\ \forall t\in T; \quad \text{(A.1k)}$$

$$gs_b{}^t \le \sum_{g\in G_b} p_g{}^t, \quad \forall b\in B,\ \forall t\in T; \quad \text{(A.1l)}$$

$$\sum_{g\in G_b} p_g{}^t + \sum_{w\in W_b} g_w{}^t + \sum_{p\in P_b} pd_p{}^t + \sum_{l\in L_b{}^{in}} f_l{}^t + ls_b{}^t = \sum_{d\in D_b} d_d{}^t + \sum_{l\in L_b{}^{out}} f_l{}^t + \sum_{p\in P_b} pc_p{}^t + ps_b{}^t, \quad \forall b\in B,\ \forall t\in T; \quad \text{(A.1m)}$$

$$w_{gi}{}^t \ge 0, \quad \forall g\in G, i\in I,\ \forall t\in T; \quad \text{(A.1n)}$$

$$-F_l \le f_l \le F_l, \quad \forall l\in L,\ \forall t\in T; \quad \text{(A.1o)}$$

$$0 \le pc_p{}^t \le Cap_p{}^{chg}, \quad \forall p\in P,\ \forall t\in T; \quad \text{(A.1p)}$$

$$0 \le pd_p{}^t \le Cap_p{}^{disc}, \quad \forall p\in P,\ \forall t\in T; \quad \text{(A.1q)}$$

$$S_p{}^{min} \le s_p{}^t \le S_p{}^{max}, \quad \forall p\in P,\ \forall t\in T. \quad \text{(A.1r)}$$

The subproblems in the MSUC formulation are indexed in $t\in T$, and Constraints (5b)–(5r) for each of these subproblems correspond exactly to the constraints in (A.1b)–(A.1r) for that $t\in T$. The subproblem for the two-stage formulation has the constraints for all the time periods together. The objective of the subproblem of the two-stage formulation is the sum over $t\in T$ of the objectives of the subproblems of the MSUC formulation, eliminating the future cost function.

References

Aminifar F, Fotuhi-Firuzabad M, Shahidehpour M (2009) Unit commitment with probabilistic spinning reserve and interruptible load considerations. *IEEE Trans. Power Systems* 24(1): 388–397.

Asamov T, Powell WB (2018) Regularized decomposition of high-dimensional multistage stochastic programs with markov uncertainty. *SIAM J. Optim.* 28(1):575–595.

BBC News (2020) Britain goes coal free as renewables edge out fossil fuels. Accessed March 13, 2021, https://www.bbc.com/news/science-environment-52973089.

Coordinador Eléctrico Nacional (2021) Estudio de control de frecuencia y determinación de reservas. Accessed July 11, 2021, https://www.coordinador.cl/wp-content/uploads/2021/06/Estudio-CFyDR-2021-Parte-1-Informe-Final.pdf.

Cordera F, Moreno R, Ordoñez F (2023) Data repository for the Chilean electricity system. Accessed February 11, 2023, https://tinyurl.com/4b78mk6f.

Cordera F, Olivares M, Ordoñez F (2021). An importance sampling approach for stochastic dual dynamic programming with correlated uncertainty. Accessed March 31, 2021, http://www.dii.uchile.cl/~fordon/docs/Corderaetal_IS_SDDP.pdf.

De Queiroz AR, Morton DP (2013) Sharing cuts under aggregated forecasts when decomposing multi-stage stochastic programs. *Oper. Res. Lett.* 41(3):311–316.

edie (2021) UK's coal power spikes amid cold snap, despite 'greenest year on record' for electricity in 2020. Accessed March 13, 2021, https://bit.ly/3fkzWOv.

Glynn PW, Iglehart DL (1989) Importance sampling for stochastic simulations. *Management Sci.* 35(11):1367–1392.

Guo Z, Wei W, Shahidehpour M, Wang Z, Mei S (2022) Optimisation methods for dispatch and control of energy storage with renewable integration. *IET Smart Grid* 5(3):137–160.

Illinois Institute of Technology (2003) Index of data. Accessed August 30, 2020, https://motor.ece.iit.edu/data/.

Infanger G, Morton DP (1996) Cut sharing for multistage stochastic linear programs with interstage dependency. *Math. Programming* 75(2):241–256.

Jiang R, Wang J, Guan Y (2011) Robust unit commitment with wind power and pumped storage hydro. *IEEE Trans. Power Systems* 27(2):800–810.

Löhndorf N, Shapiro A (2019) Modeling time-dependent randomness in stochastic dual dynamic programming. *Eur. J. Oper. Res.* 273(2):650–661.

Ministerio de Energía (2021) Gobierno de Chile. Accessed March 15, 2021, https://energia.gob.cl/pelp/proyeccion-y-emisiones.

Morales-España G, Lorca Á, de Weerdt MM (2018) Robust unit commitment with dispatchable wind power. *Electric Power Systems Res.* 155:58–66.

Papavasiliou A, Oren SS (2013) Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network. *Oper. Res.* 61(3):578–592.

Papavasiliou A, Mou Y, Cambier L, Scieur D (2017) Application of stochastic dual dynamic programming to the real-time dispatch of storage under renewable supply uncertainty. *IEEE Trans. Sustainable Energy* 9(2):547–558.

Pereira MV (1989) Optimal stochastic operations scheduling of large hydroelectric systems. *Internat. J. Electric Power Energy Systems* 11(3):161–169.

Philpott AB, De Matos VL (2012) Dynamic sampling algorithms for multi-stage stochastic programs with risk aversion. *Eur. J. Oper. Res.* 218(2):470–483.

Pinson P (2013) Wind energy: Forecasting challenges for its operational management. *Statist. Sci.* 28(4):564–585.

PJM (2020) PJM solar forecast 2020. Accessed March 13, 2021, https://www.pjm.com/-/media/committees-groups/subcommittees/

las/2020/20201130/20201130-item-03a-ihs-markit-pjm-solar-forecast.ashx.

Ruiz PA, Philbrick CR, Zak E, Cheung KW, Sauer PW (2009) Uncertainty management in the unit commitment problem. *IEEE Trans. Power Systems* 24(2):642–651.

Shapiro A (2003) Inference of statistical bounds for multistage stochastic programming problems. *Math. Methods Oper. Res.* 58(1):57–68.

Shapiro A, Tekaya W, da Costa JP, Soares MP (2013) Risk neutral and risk averse stochastic dual dynamic programming method. *Eur. J. Oper. Res.* 224(2):375–391.

Shiina T, Birge JR (2004) Stochastic unit commitment problem. *Internat. Trans. Oper. Res.* 11(1):19–32.

Silva V (2010) Value of flexibility in systems with large wind penetration. PhD dissertation, Imperial College London, London.

Sioshansi R, O'Neill R, Oren SS (2008) Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets. *IEEE Trans. Power Systems* 23(2):344–352.

Tokdar ST, Kass RE (2010) Importance sampling: A review. *Wiley Interdisciplinary Rev. Comput. Statist.* 2(1):54–60.

Uçkun C, Botterud A, Birge JR (2015) An improved stochastic unit commitment formulation to accommodate wind uncertainty. *IEEE Trans. Power Systems* 31(4):2507–2517.

Xiong P, Jirutitijaroen P, Singh C (2016) A distributionally robust optimization model for unit commitment considering uncertain wind power generation. *IEEE Trans. Power Systems* 32(1):39–49.

Zéphyr L, Anderson CL (2018) Stochastic dynamic programming approach to managing power system uncertainty with distributed storage. *Comput. Managment Sci.* 15(1):87–110.

Zheng QP, Wang J, Liu AL (2014) Stochastic optimization for unit commitment—A review. *IEEE Trans. Power Systems* 30(4):1913–1924.

Zou J, Ahmed S, Sun XA (2018) Multistage stochastic unit commitment using stochastic dual dynamic integer programming. *IEEE Trans. Power Systems* 34(3):1814–1823.

Zou J, Ahmed S, Sun XA (2019) Stochastic dual dynamic integer programming. *Math. Programming* 175(1):461–502.

Felipe Cordera is a PhD student at the Operations Research Center, Massachusetts Institute of Technology. His research focuses on applying mathematical programming techniques to solve complex optimization problems, particularly those affected by uncertainty and high dimensionality, with a keen interest in decomposition methods and modeling aspects.

Rodrigo Moreno is with the electrical engineering department at the University of Chile and Instituto Sistemas Complejos de Ingeniería. His research interests involve power systems operations and planning under uncertainty, electricity markets, policy, and regulation.

Fernando Ordoñez is a professor in the industrial engineering department at University of Chile. His research interests involve the theory and applications of optimization.